

The Statistical Sleuth in R:

Chapter 10

Kate Aloisio Ruobing Zhang Nicholas J. Horton*

August 24, 2024

Contents

| | | |
|----------|---|----------|
| 1 | Introduction | 1 |
| 2 | Galileo’s data on the motion of falling bodies | 2 |
| 2.1 | Data coding, summary statistics and graphical display | 2 |
| 2.2 | Models | 3 |
| 3 | Echolocation in bats | 6 |
| 3.1 | Data coding, summary statistics and graphical display | 6 |
| 3.2 | Multiple regression | 8 |

1 Introduction

This document is intended to help describe how to undertake analyses introduced as examples in the Second Edition of the *Statistical Sleuth* (2002) by Fred Ramsey and Dan Schafer. More information about the book can be found at <http://www.proaxis.com/~panorama/home.htm>. This file as well as the associated `knitr` reproducible analysis source file can be found at <http://www.amherst.edu/~nhorton/sleuth>.

This work leverages initiatives undertaken by Project MOSAIC (<http://www.mosaic-web.org>), an NSF-funded effort to improve the teaching of statistics, calculus, science and computing in the undergraduate curriculum. In particular, we utilize the `mosaic` package, which was written to simplify the use of R for introductory statistics courses. A short summary of the R needed to teach introductory statistics can be found in the `mosaic` package vignette (<http://cran.r-project.org/web/packages/mosaic/vignettes/MinimalR.pdf>).

To use a package within R, it must be installed (one time), and loaded (each session). The package can be installed using the following command:

```
> install.packages('mosaic') # note the quotation marks
```

Once this is installed, it can be loaded by running the command:

*Department of Mathematics, Amherst College, nhorton@amherst.edu

```
> require(mosaic)
```

This needs to be done once per session.

In addition the data files for the *Sleuth* case studies can be accessed by installing the **Sleuth2** package.

```
> install.packages('Sleuth2') # note the quotation marks
```

```
> require(Sleuth2)
```

We also set some options to improve legibility of graphs and output.

```
> trellis.par.set(theme=col.mosaic()) # get a better color scheme for lattice
> options(digits=3)
```

The specific goal of this document is to demonstrate how to calculate the quantities described in Chapter 10: Inferential Tools for Multiple Regression using R.

2 Galileo's data on the motion of falling bodies

Galileo investigated the relationship between height and horizontal distance. This is the question addressed in case study 10.1 in the *Sleuth*.

2.1 Data coding, summary statistics and graphical display

We begin by reading the data and summarizing the variables.

```
> summary(case1001)
```

| Distance | Height |
|-------------|--------------|
| Min. :253 | Min. : 100 |
| 1st Qu.:366 | 1st Qu.: 250 |
| Median :451 | Median : 450 |
| Mean :434 | Mean : 493 |
| 3rd Qu.:514 | 3rd Qu.: 700 |
| Max. :573 | Max. :1000 |

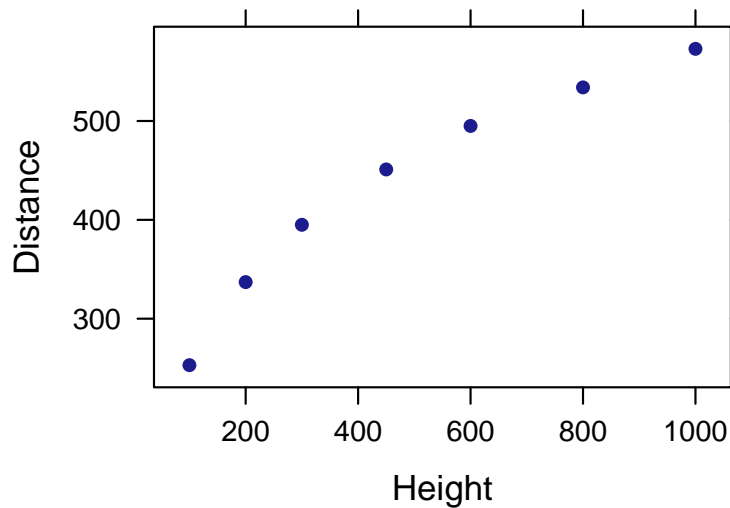
```
> favstats(~ Distance, data=case1001)
```

| min | Q1 | median | Q3 | max | mean | sd | n | missing |
|-----|-----|--------|-----|-----|------|-----|---|---------|
| 253 | 366 | 451 | 514 | 573 | 434 | 113 | 7 | 0 |

There we a total of 7 trials of Galileo's experiment. For each trial, he recorded the initial height and then measured the horizontal distance as shown in Display 10.1 (page 268).

We can start to explore this relationship by creating a scatterplot of Galileo's horizontal distances versus initial heights. The following graph is akin to Display 10.2 (page 269).

```
> xyplot(Distance ~ Height, data=case1001)
```



2.2 Models

The first model that we created is a cubic model as interpreted on page 269 and summarized in Display 10.13 (page 286).

```
> lm1 = lm(Distance ~ Height+I(Height^2)+I(Height^3), data=case1001); summary(lm1)
```

Call:

```
lm(formula = Distance ~ Height + I(Height^2) + I(Height^3), data = case1001)
```

Residuals:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---------|--------|--------|---------|---------|--------|---------|
| -2.4036 | 3.5809 | 1.8917 | -4.4688 | -0.0804 | 2.3216 | -0.8414 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-------------|-----------|------------|---------|----------|
| (Intercept) | 1.56e+02 | 8.33e+00 | 18.71 | 0.00033 |
| Height | 1.12e+00 | 6.57e-02 | 16.98 | 0.00044 |
| I(Height^2) | -1.24e-03 | 1.38e-04 | -8.99 | 0.00290 |
| I(Height^3) | 5.48e-07 | 8.33e-08 | 6.58 | 0.00715 |

Residual standard error: 4.01 on 3 degrees of freedom

Multiple R-squared: 0.999, Adjusted R-squared: 0.999

F-statistic: 1.6e+03 on 3 and 3 DF, p-value: 2.66e-05

We next decrease the polynomial for *Height* by one degree to obtain a quadratic model as interpreted on page 269 and summarized in Display 10.7 (page 277). This model is used for most of the following results.

```
> lm2 = lm(Distance ~ Height+I(Height^2), data=case1001); summary(lm2)
```

Call:

```
lm(formula = Distance ~ Height + I(Height^2), data = case1001)
```

Residuals:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|--------|------|-------|------|-------|--------|------|
| -14.31 | 9.17 | 13.52 | 1.94 | -6.18 | -12.61 | 8.46 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-------------|-----------|------------|---------|----------|
| (Intercept) | 2.00e+02 | 1.68e+01 | 11.93 | 0.00028 |
| Height | 7.08e-01 | 7.48e-02 | 9.47 | 0.00069 |
| I(Height^2) | -3.44e-04 | 6.68e-05 | -5.15 | 0.00676 |

Residual standard error: 13.6 on 4 degrees of freedom

Multiple R-squared: 0.99, Adjusted R-squared: 0.986

F-statistic: 205 on 2 and 4 DF, p-value: 9.33e-05

We can fit an equivalent model (with different parametrization) using the `poly()` function:

```
> lmpoly = lm(Distance ~ poly(Height, 2), data=case1001); summary(lmpoly)
```

Call:

```
lm(formula = Distance ~ poly(Height, 2), data = case1001)
```

Residuals:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|--------|------|-------|------|-------|--------|------|
| -14.31 | 9.17 | 13.52 | 1.94 | -6.18 | -12.61 | 8.46 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|------------------|----------|------------|---------|----------|
| (Intercept) | 434.00 | 5.16 | 84.19 | 1.2e-07 |
| poly(Height, 2)1 | 267.12 | 13.64 | 19.58 | 4.0e-05 |
| poly(Height, 2)2 | -70.19 | 13.64 | -5.15 | 0.0068 |

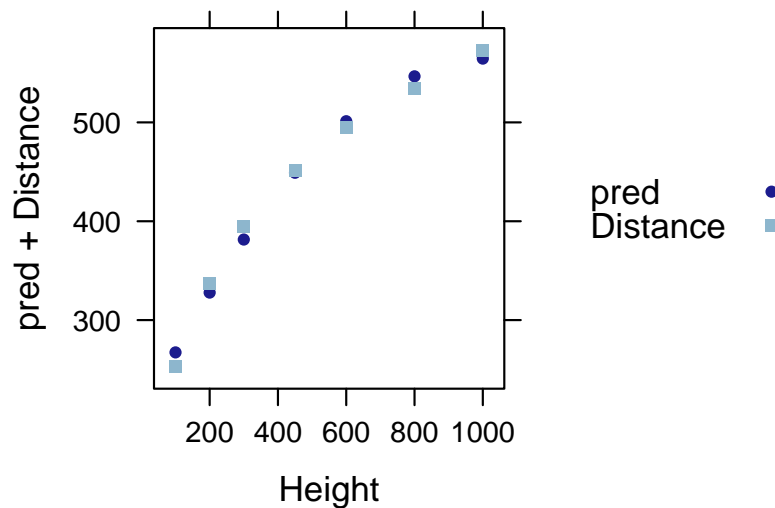
Residual standard error: 13.6 on 4 degrees of freedom

Multiple R-squared: 0.99, Adjusted R-squared: 0.986

F-statistic: 205 on 2 and 4 DF, p-value: 9.33e-05

The following figure presents the predicted values from the quadratic model and the original data points akin to Display 10.2 (page 269).

```
> case1001 = transform(case1001, pred = predict(lm2))
> xyplot(pred+Distance ~ Height, auto.key=TRUE, data=case1001)
```



To obtain the expected values of $\hat{\mu}(\text{Distance}|\text{Height} = 0)$ and $\hat{\mu}(\text{Distance}|\text{Height} = 250)$, we used the `predict()` command with the quadratic model as shown in Display 10.7 (page 277).

```
> predict(lm2, interval="confidence", data.frame(Height=c(0, 250)))

fit lwr upr
1 200 153 246
2 356 337 374
```

We can also verify the above confidence interval calculations with the following code:

```
> 355.1+c(-1, 1)*6.62*qt(.975, 4)

[1] 337 373
```

To verify numbers on page 279, an interval for the predicted values, we used the following code:

```
> predict(lm2, interval="predict", data.frame(Height=c(0, 250)))

fit lwr upr
1 200 140 260
2 356 313 398
```

Lastly, we display the ANOVA table for the quadratic model that is interpreted on page 283 (Display 10.11).

```
> anova(lm2)

Analysis of Variance Table

Response: Distance
          Df Sum Sq Mean Sq F value Pr(>F)
Height      1  71351   71351   383.6 4e-05
I(Height^2) 1   4927    4927    26.5 0.0068
Residuals   4    744     186
```

3 Echolocation in bats

How do bats make their way about in the dark? Echolocation requires a lot of energy. Does it depend on mass and species? This is the question addressed in case study 10.2 in the *Sleuth*.

3.1 Data coding, summary statistics and graphical display

We begin by reading the data, performing transformations where necessary and summarizing the variables.

```
> case1002 = transform(case1002, logmass = log(Mass))
> case1002 = transform(case1002, logenergy = log(Energy))
> summary(case1002)
```

| Species | Mass | Type | Energy |
|------------------|--------------|---------------------------|--------------|
| Length:20 | Min. : 7 | non-echolocating bats : 4 | Min. : 1.0 |
| Class :character | 1st Qu.: 63 | non-echolocating birds:12 | 1st Qu.: 7.6 |
| Mode :character | Median :266 | echolocating bats : 4 | Median :22.6 |
| | Mean :263 | | Mean :19.5 |
| | 3rd Qu.:391 | | 3rd Qu.:28.2 |
| | Max. :779 | | Max. :43.7 |
| logmass | logenergy | | |
| Min. :1.90 | Min. :0.02 | | |
| 1st Qu.:4.10 | 1st Qu.:1.98 | | |
| Median :5.58 | Median :3.12 | | |
| Mean :4.89 | Mean :2.48 | | |
| 3rd Qu.:5.97 | 3rd Qu.:3.34 | | |
| Max. :6.66 | Max. :3.78 | | |

```
> favstats(Mass ~ Type, data=case1002)
```

```

      Type  min    Q1 median    Q3 max  mean   sd  n missing
1 non-echolocating bats 258.0 300.75 471.50 665.8 779 495.0 249.6 4      0
2 non-echolocating birds 24.3 108.20 302.50 391.0 480 263.2 165.2 12     0
3   echolocating bats   6.7   7.45   7.85 29.2 93 28.8 42.8 4      0

> favstats(Energy ~ Type, data=case1002)

      Type  min    Q1 median    Q3  max  mean   sd  n missing
1 non-echolocating bats 22.40 23.07 29.05 37.02 43.70 31.05 10.15 4      0
2 non-echolocating birds 2.46 12.64 24.35 28.22 43.70 21.15 12.52 12     0
3   echolocating bats 1.02 1.09 1.24 3.22 8.83 3.08 3.84 4      0

```

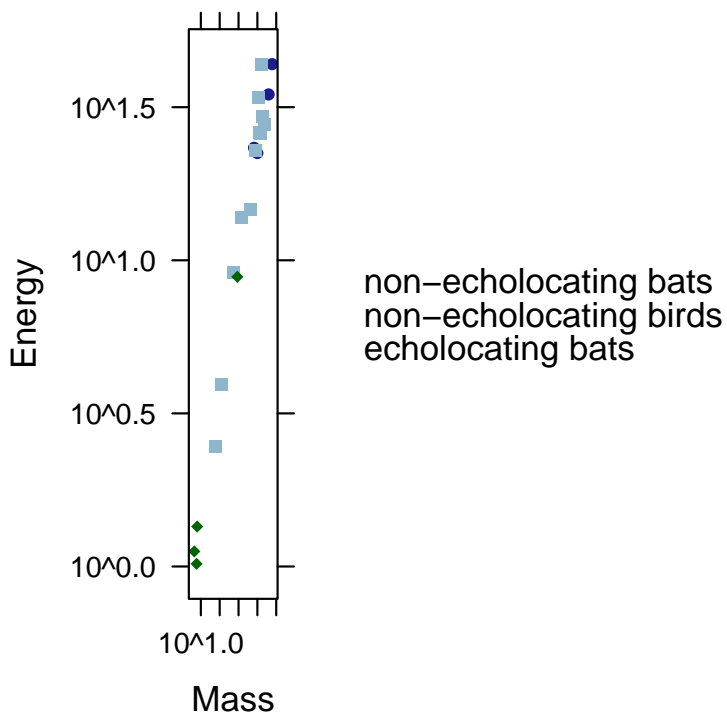
A total of 20 flying vertebrates were included in this study. There were 4 echolocating bats, 4 non-echolocating bats, and 12 non-echolocating birds. For each subject their *mass* and *flight energy expenditure* were recorded as shown in Display 10.3 (page 270).

We can next observe the pattern between log(energy expenditure) as a function of log(body mass) for each group with a scatterplot. The following figure is akin to Display 10.4 (page 271).

```

> xyplot(Energy ~ Mass, group=Type, scales=list(y=list(log=TRUE),
+       x=list(log=TRUE)), auto.key=TRUE, data=case1002)

```



3.2 Multiple regression

We first evaluate a multiple regression model for $\log(\text{energy expenditure})$ given type of species and $\log(\text{body mass})$ as defined on page 272 and shown in Display 10.6 (page 273).

```
> lm1 = lm(logenergy ~ logmass+Type, data=case1002); summary(lm1)
```

Call:
lm(formula = logenergy ~ logmass + Type, data = case1002)

Residuals:

| Min | 1Q | Median | 3Q | Max |
|---------|---------|---------|--------|--------|
| -0.2322 | -0.1220 | -0.0364 | 0.1257 | 0.3446 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|----------------------------|----------|------------|---------|----------|
| (Intercept) | -1.5764 | 0.2872 | -5.49 | 5.0e-05 |
| logmass | 0.8150 | 0.0445 | 18.30 | 3.8e-12 |
| Typenon-echolocating birds | 0.1023 | 0.1142 | 0.90 | 0.38 |
| Typeecholocating bats | 0.0787 | 0.2027 | 0.39 | 0.70 |

Residual standard error: 0.186 on 16 degrees of freedom
Multiple R-squared: 0.982, Adjusted R-squared: 0.978
F-statistic: 284 on 3 and 16 DF, p-value: 4.46e-14

Next, we calculated confidence intervals for the coefficients which are interpreted on page 274.

```
> confint(lm1)
```

| | 2.5 % | 97.5 % |
|----------------------------|--------|--------|
| (Intercept) | -2.185 | -0.967 |
| logmass | 0.721 | 0.909 |
| Typenon-echolocating birds | -0.140 | 0.344 |
| Typeecholocating bats | -0.351 | 0.508 |

```
> exp(confint(lm1))
```

| | 2.5 % | 97.5 % |
|----------------------------|-------|--------|
| (Intercept) | 0.112 | 0.38 |
| logmass | 2.056 | 2.48 |
| Typenon-echolocating birds | 0.870 | 1.41 |
| Typeecholocating bats | 0.704 | 1.66 |

Since the significance of a model depends on which variables are included, the *Sleuth* proposes two other models, one only looking at the type of flying animal and the other allows the three

groups to have different straight-line regressions with *mass*. These two models are displayed below and discussed on pages 274–275.

```
> summary(lm(logenergy ~ Type, data=case1002))
```

Call:
lm(formula = logenergy ~ Type, data = case1002)

Residuals:

| Min | 1Q | Median | 3Q | Max |
|---------|---------|--------|--------|--------|
| -1.8872 | -0.3994 | 0.0236 | 0.4932 | 1.5253 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|--|----------|------------|---------|----------|
| (Intercept) | 3.396 | 0.422 | 8.04 | 3.4e-07 |
| Type _{non-echolocating birds} | -0.609 | 0.488 | -1.25 | 0.22885 |
| Type _{echolocating bats} | -2.743 | 0.597 | -4.59 | 0.00026 |

Residual standard error: 0.845 on 17 degrees of freedom
Multiple R-squared: 0.595, Adjusted R-squared: 0.548
F-statistic: 12.5 on 2 and 17 DF, p-value: 0.000458

```
> summary(lm(logenergy ~ Type * logmass, data=case1002))
```

Call:
lm(formula = logenergy ~ Type * logmass, data = case1002)

Residuals:

| Min | 1Q | Median | 3Q | Max |
|---------|---------|---------|--------|--------|
| -0.2515 | -0.1264 | -0.0095 | 0.0812 | 0.3284 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|---|----------|------------|---------|----------|
| (Intercept) | -0.202 | 1.261 | -0.16 | 0.875 |
| Type _{non-echolocating birds} | -1.378 | 1.295 | -1.06 | 0.305 |
| Type _{echolocating bats} | -1.268 | 1.285 | -0.99 | 0.341 |
| logmass | 0.590 | 0.206 | 2.86 | 0.013 |
| Type _{non-echolocating birds} :logmass | 0.246 | 0.213 | 1.15 | 0.269 |
| Type _{echolocating bats} :logmass | 0.215 | 0.224 | 0.96 | 0.353 |

Residual standard error: 0.19 on 14 degrees of freedom
Multiple R-squared: 0.983, Adjusted R-squared: 0.977
F-statistic: 163 on 5 and 14 DF, p-value: 6.7e-12

To construct the confidence bands discussed on page 277 and shown in Display 10.9 (page 279) we used the following code:

```
> pred = predict(lm1, se.fit=TRUE, newdata=data.frame(Type=c("non-echolocating birds", "non-echolocating bats")))
> pred.fit = pred$fit[1]; pred.fit

      1
2.28

> pred.se = pred$se.fit[1]; pred.se

      1
0.0604

> multiplier = sqrt(4*qf(.95, 4, 16)); multiplier

[1] 3.47

> lower = exp(pred.fit-pred.se*multiplier); lower

      1
7.92

> upper = exp(pred.fit+pred.se*multiplier); upper

      1
12

> # for the other reference points
> pred2 = predict(lm1, se.fit=TRUE, newdata=data.frame(Type=c("non-echolocating bats", "non-echolocating birds")))
> pred3 = predict(lm1, se.fit=TRUE, newdata=data.frame(Type=c("echolocating bats", "echolocating birds")))
>
> table10.9 = rbind(c("Intercept estimate", "Standard error"), round(cbind(pred2$fit, pred2$se), 4))

      [,1]          [,2]
"Intercept estimate" "Standard error"
1 "2.1767"          "0.1144"
2 "3.3064"          "0.0931"
1 "2.2553"          "0.1277"
2 "3.3851"          "0.1759"
```

Next we can assess the model by evaluating the extra sums of squares F -test for testing the equality of intercepts in the parallel regression lines model as shown in Display 10.10 (page 282).

```
> lm2 = lm(logenergy ~ logmass, data=case1002)
> anova(lm2, lm1)
```

Analysis of Variance Table

```

Model 1: logenergy ~ logmass
Model 2: logenergy ~ logmass + Type
  Res.Df  RSS Df Sum of Sq   F Pr(>F)
1     18 0.583
2     16 0.553  2    0.0296 0.43  0.66

```

We can also compare the full model with interaction terms and the reduced model (without interaction terms) with the extra sum of squares F -test as described in Display 10.12 (page 284).

```

> lm3 = lm(logenergy ~ logmass*Type, data=case1002)
> anova(lm3, lm1)

```

Analysis of Variance Table

```

Model 1: logenergy ~ logmass * Type
Model 2: logenergy ~ logmass + Type
  Res.Df  RSS Df Sum of Sq   F Pr(>F)
1     14 0.505
2     16 0.553 -2   -0.0484 0.67  0.53

```

Another way to test the equality of the groups is by using linear combinations which we can attain using the `estimable()` command as follows. These results can be found on page 276 and 289.

```

> require(gmodels)
> estimable(lm1, c(0, 0, -1, 1))

      Estimate Std. Error t value DF Pr(>|t|)
(0 0 -1 1) -0.0236    0.158   -0.15 16  0.883

```