The Statistical Sleuth in R: Chapter 6

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1 Introduction

This document is intended to help describe how to undertake analyses introduced as examples in the Third Edition of the *Statistical Sleuth* (2013) by Fred Ramsey and Dan Schafer. More information about the book can be found at http://www.proaxis.com/~panorama/home.htm. This file as well as the associated knitr reproducible analysis source file can be found at http://www.math.smith.edu/~nhorton/sleuth3.

This work leverages initiatives undertaken by Project MOSAIC (http://www.mosaic-web. org), an NSF-funded effort to improve the teaching of statistics, calculus, science and computing in the undergraduate curriculum. In particular, we utilize the mosaic package, which was written to simplify the use of R for introductory statistics courses. A short summary of the R needed to teach introductory statistics can be found in the mosaic package vignette (http://cran.r-project. org/web/packages/mosaic/vignettes/MinimalR.pdf).

To use a package within R, it must be installed (one time), and loaded (each session). The package can be installed using the following command:

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2 DISCRIMINATION AGAINST THE HANDICAPPED

```
> install.packages('mosaic')  # note the quotation marks
```

Once this is installed, it can be loaded by running the command:

> require(mosaic)

This needs to be done once per session.

In addition the data files for the *Sleuth* case studies can be accessed by installing the **Sleuth3** package.

```
> install.packages('Sleuth3')
```

note the quotation marks

> require(Sleuth3)

We also set some options to improve legibility of graphs and output.

```
> trellis.par.set(theme=col.mosaic()) # get a better color scheme for lattice
> options(digits=3)
```

The specific goal of this document is to demonstrate how to calculate the quantities described in Chapter 6: Linear Combinations and Multiple Comparisons of Means using R.

2 Discrimination Against the Handicapped

Do equivalent candidates with the same qualifications but different disabilities get treated differentially? This is the question addressed in case study 6.1 in the *Sleuth*.

2.1 Summary statistics and graphical display

We begin by reading the data and summarizing the variables.

```
> case0601$Handicap = relevel(case0601$Handicap, ref="Amputee")
> summary(case0601)
                     Handicap
    Score
Min. :1.40 Amputee
                        :14
 1st Qu.:3.70 Crutches :14
Median :5.05
               Hearing
                         :14
Mean :4.93
               None
                         :14
 3rd Qu.:6.10
               Wheelchair:14
Max.
       :8.50
> favstats(Score ~ Handicap, data=case0601)
```

	Handicap	min	Q1	median	Q3	\max	mean	sd	n	missing	
1	Amputee	1.9	3.30	4.30	5.72	7.2	4.43	1.59	14	0	
2	Crutches	3.7	4.50	6.10	7.15	8.5	5.92	1.48	14	0	
З	Hearing	1.4	3.02	4.05	5.30	6.5	4.05	1.53	14	0	
4	None	1.9	3.73	5.00	6.05	7.8	4.90	1.79	14	0	
5	Wheelchair	1.7	4.73	5.70	6.35	7.6	5.34	1.75	14	0	

A total of 70 undergraduate students from a U.S. university were randomly assigned to view the tapes, 14 to each tape. The five kinds of tapes are: *None*, *Amputee*, *Crutches*, *Hearing* and *Wheelchair*. After reviewing the tape, each subject rated the qualifications of the application on 0-10 scale. Among the five handicap conditions, the *Crutches* group gave the highest mean score, while the *Hearing* group gave the lowest mean score. This is summarized on page 150 and in Display 6.1 of the *Sleuth*.

```
> with(subset(case0601, Handicap=="None"), stem(Score, scale=2))
 The decimal point is at the |
  1 | 9
  2 | 5
  3 | 06
  4 | 129
  5 | 149
  6 | 17
 7 | 48
> with(subset(case0601, Handicap=="Amputee"), stem(Score, scale=2))
 The decimal point is at the |
  1 | 9
  2 | 56
  3 | 268
 4 | 06
  5 | 3589
  6 | 1
 7 | 2
> with(subset(case0601, Handicap=="Crutches"), stem(Score, scale=1))
 The decimal point is at the |
 3 | 7
```

```
4 | 033
 5 | 18
 6 | 0234
 7 | 445
 8 | 5
> with(subset(case0601, Handicap=="Hearing"), stem(Score, scale=2))
 The decimal point is at the |
 1 | 4
  2 | 149
 3 | 479
 4 | 237
 5 | 589
 6 | 5
> with(subset(case0601, Handicap=="Wheelchair"), stem(Score, scale=2))
 The decimal point is at the |
 1 | 7
 2 | 8
 3 | 5
 4 | 78
 5 | 03
 6 | 1124
7 | 246
```

```
> bwplot(Handicap ~ Score, data=case0601)
```



> densityplot(~ Score, groups=Handicap, auto.key=TRUE, data=case0601)



The stem plots show the applicant qualification scores given by objectives. The boxplots and the density plots show that all the distributions are approximately normally distributed.

2.2 One-way ANOVA

First we fit the one way analysis of variance (ANOVA) model, using all of the groups. This corresponds to the interpretations on page 151.

> anova(lm(Score ~ Handicap, data=case0601))

Analysis of Variance Table

Response: Score Df Sum Sq Mean Sq F value Pr(>F) Handicap 4 30.5 7.63 2.86 0.03 Residuals 65 173.3 2.67

The p-value provides some evidence that subjects rate qualifications differently according to handicap status.

By default, the use of the linear model (regression) function displays the pairwise differences between the first group and each of the other groups. Note that the overall test of the model is the same.

> summary(lm(Score ~ Handicap, data=case0601)) Call: lm(formula = Score ~ Handicap, data = case0601) Residuals: Min 1Q Median ЗQ Max -3.643 -1.209 0.114 1.329 2.900 Coefficients: Estimate Std. Error t value Pr(>|t|) (Intercept) 4.429 0.436 10.15 5e-15 HandicapCrutches 1.493 0.617 2.42 0.018 HandicapHearing -0.379 0.617 -0.61 0.542 HandicapNone 0.471 0.617 0.76 0.448 HandicapWheelchair 0.914 0.617 1.48 0.143 Residual standard error: 1.63 on 65 degrees of freedom Multiple R-squared: 0.15, Adjusted R-squared: 0.0974 F-statistic: 2.86 on 4 and 65 DF, p-value: 0.0301

The reference group here is *Amputee*, followed by *Crutches*, *Hearing*, *None* and *Wheelchair*. Another way of viewing these results is through a model table, which displays the differences between the grand mean and the group means.

```
> model.tables(aov(Score ~ Handicap, data=case0601))
Tables of effects
Handicap
Amputee Crutches Hearing None Wheelchair
-0.5000 0.9929 -0.8786 -0.0286 0.4143
```

Or by:

```
> mean(Score ~ Handicap, data=case0601)-mean(~ Score, data=case0601)
Amputee Crutches Hearing None Wheelchair
-0.5000 0.9929 -0.8786 -0.0286 0.4143
```

2.3 Contrasts and linear combination

The Tukey-Kramer test is a reasonable method for these data. We can use this to verify the calculation on page 151.

```
> TukeyHSD(aov(lm(Score ~ Handicap, data=case0601)), "Handicap", ordered=TRUE, c(0,1,-1,0,0),
 Tukey multiple comparisons of means
   95% family-wise confidence level
   factor levels have been ordered
Fit: aov(formula = lm(Score ~ Handicap, data = case0601))
$Handicap
                            lwr upr p adj
                    diff
                   0.379 -1.353 2.11 0.972
Amputee-Hearing
None-Hearing
                   0.850 -0.882 2.58 0.644
Wheelchair-Hearing 1.293 -0.439 3.02 0.235
Crutches-Hearing
                  1.871 0.140 3.60 0.028
None-Amputee
                   0.471 -1.260 2.20 0.940
Wheelchair-Amputee 0.914 -0.817 2.65 0.578
Crutches-Amputee
                  1.493 -0.239 3.22 0.123
Wheelchair-None
                   0.443 -1.289 2.17 0.952
Crutches-None
                   1.021 -0.710 2.75 0.469
Crutches-Wheelchair 0.579 -1.153 2.31 0.881
```

Based on the Tukey-Kramer procedure, the difference is estimated to be higher for the *Crutches* tapes.

Next, we calculate the comparison of Amputee/Hearing to Crutches/Wheelchair.

The results indicate a statistically significant difference between the average scores given to the *Wheelchair* and *Crutches* handicaps and the average scores given to the *Amputee* and *Hearing* handicaps.

To verify the calculations on page 155 we used the following contrast:

Other multiple comparison procedures could also be implemented. The following shows the calculation on page 164.

```
> require(agricolae)
```

Loading required package: agricolae

```
> LSD.test(aov(lm(Score ~ Handicap, data=case0601)), "Handicap") # LSD
> HSD.test(aov(lm(Score ~ Handicap, data=case0601)), "Handicap") # Tukey-Kramer
> LSD.test(aov(lm(Score ~ Handicap, data=case0601)), "Handicap", p.adj=c("bonferroni")) # Bon
> scheffe.test(aov(lm(Score ~ Handicap, data=case0601)), "Handicap") # Scheffe
```

The "Significant Difference" in each test result is the "95% interval half-width" described in the book.

3 Pre-existing Preference of Fish

Was Charles Darwin right that sexual selection is driven by females? This is the question addressed in case study 6.2 in the *Sleuth*.

3.1 Summary statistics and graphical display

We begin by reading the data and summarizing the variables.

```
> summary(case0602)
  Percentage
                              Length
                  Pair
Min. :10.0
                          Min. :28.0
               Pair1:16
 1st Qu.:53.1
              Pair2:14
                         1st Qu.:31.0
Median :61.5
                         Median :34.0
               Pair3:17
Mean :62.1
               Pair4:14
                          Mean :32.8
 3rd Qu.:71.8
               Pair5: 9
                          3rd Qu.:34.0
Max. :92.4
               Pair6:14
                         Max. :35.0
> favstats(Percentage ~ Pair, data=case0602)
```

	Pair	min	Q1	median	QЗ	max	mean	sd	n	missing
1	Pair1	43.7	49.7	55.3	63.1	73.3	56.4	9.02	16	0
2	Pair2	39.6	53.1	64.4	69.6	80.2	60.9	12.48	14	0
3	Pair3	10.0	50.6	62.0	83.6	91.3	62.4	22.29	17	0
4	Pair4	42.0	57.2	67.9	76.2	92.4	67.0	14.33	14	0
5	Pair5	47.7	61.0	62.9	66.0	78.3	64.2	9.41	9	0
6	Pair6	33.4	56.7	62.7	78.9	87.6	63.3	17.68	14	0

A total of 84 female fish were involved in this experiment, which is shown on page 153.

> bwplot(Pair ~ Percentage, data=case0602)



> densityplot(~ Percentage, groups=Pair, auto.key=TRUE, data=case0602)



Besides the distribution of pair 5, all distributions of other pairs are approximately normally distributed.

3.2 One-way ANOVA

First we fit the one way analysis of variance (ANOVA) model, using all of the groups:

The p-value is not small, and does not provide much evidence that the mean percentage of time with the yellow-sword male significantly differed from one male pair to another back in the population.

By default, the use of the linear model (regression) function displays the pairwise differences between the first group and each of the other groups. Note that the overall test of the model is the same.

```
> summary(lm(Percentage ~ Pair, data=case0602))
Call:
lm(formula = Percentage ~ Pair, data = case0602)
Residuals:
```

Min	1Q	Median		ЗŲ	1	Max				
-52.43 -8	.41	0.25	10.8	86	28	. 87	•			
Coefficien	ts:									
	Es	stimate	Std.	Er	ror	t	value	Pr(> t))	
(Intercept))	56.41		3	.86		14.60	<2e-16	5	
PairPair2		4.48		5	.66		0.79	0.431	1	
PairPair3		6.02		5	.38		1.12	0.267	7	
PairPair4		10.59		5	.66		1.87	0.065	5	
PairPair5		7.80		6	.44		1.21	0.229	9	
PairPair6		6.93		5	.66		1.22	0.224	1	
Residual standard error: 15.5 on 78 degrees of freedom										
Multiple R	-sqi	lared:	0.048	8, 1	Adjı	ust	ed R-s	squared:	-0.01	31
F-statisti	c: ().786 or	15 ai	nd '	78 1	DF,	p-va	alue: 0.5	563	

The reference group here is pair 1, followed by pairs 2-6. Another way of viewing these results is through a model table, which displays the differences between the grand mean and the group means.

> model.tables(aov(Percentage ~ Pair, data=case0602))

Tables of effects

Pair

Pair1 Pair2 Pair3 Pair4 Pair5 Pair6 -5.722 -1.243 0.3008 4.871 2.083 1.207 rep 16.000 14.000 17.0000 14.000 9.000 14.000

Or by:

```
> mean(Percentage ~ Pair, data=case0602)-mean(~ Percentage, data=case0602)
Pair1 Pair2 Pair3 Pair4 Pair5 Pair6
-5.722 -1.243 0.301 4.871 2.083 1.207
```

3.3 Contrasts and linear combination

We can calculate the values on page 152 and Display 6.5 on page 158 using contrasts.

```
> t=round(lc[, "t value"], 2); t
```

[1] -0.46

```
> pt(t, 78, lower.tail=TRUE)
```

[1] 0.323

The *t*-value is -0.46 and the one-sided *p*-value is 0.32.

```
> mean(mean(Percentage ~ Pair, data=case0602))
[1] 62.4
> t.test(mean(Percentage ~ Pair, data=case0602))
One Sample t-test
data: mean(Percentage ~ Pair, data = case0602)
t = 43, df = 5, p-value = 1e-07
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
    58.6 66.1
sample estimates:
mean of x
    62.4
```

The estimated mean percentage of time spent with the yellow-sword male is 62.378%. The one-sided *p*-value < 0.0001, and the 95% confidence interval is (58.637%, 66.119%).