

# Package: copula (via r-universe)

August 19, 2024

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**Title** Multivariate Dependence with Copulas

**Depends** R (>= 3.5.0)

**Imports** stats, graphics, methods, stats4, Matrix (>= 1.5-0), lattice, colorspace, gsl, ADGofTest, stabledist (>= 0.6-4), mvtnorm, pcaPP, pspline, numDeriv

**Suggests** MASS, KernSmooth, sfsmisc, scatterplot3d, Rmpfr, bbmle, knitr, rmarkdown, abind, crop, gridExtra, lcopula, mev, mvnormttest, parallel, partitions, polynom, qrng, randtoolbox, rugarch, Runuran, tseries, VGAM, VineCopula, zoo

**SuggestsNote** packages {abind, ..., zoo} (last lines above) are only used in vignettes, demos and a few tests.

**VignetteBuilder** knitr

**Enhances** nor1mix, copulaData

**SystemRequirements** pdfcrop (part of TexLive) is required to rebuild the vignettes.

**Description** Classes (S4) of commonly used elliptical, Archimedean, extreme-value and other copula families, as well as their rotations, mixtures and asymmetrizations. Nested Archimedean copulas, related tools and special functions. Methods for density, distribution, random number generation, bivariate dependence measures, Rosenblatt transform, Kendall distribution function, perspective and contour plots. Fitting of copula models with potentially partly fixed parameters, including standard errors. Serial independence tests, copula specification tests (independence, exchangeability, radial symmetry, extreme-value dependence, goodness-of-fit) and model selection based on cross-validation. Empirical copula, smoothed versions, and non-parametric estimators of the Pickands dependence function.

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**Collate** AllClass.R Classes.R AllGeneric.R Auxiliaries.R aux-acopula.R  
 asymCopula.R mixCopula.R rotCopula.R Copula.R special-func.R  
 amhCopula.R claytonCopula.R frankCopula.R cop\_objects.R  
 nacopula.R dC-dc.R amhExpr.R An.R archmCopula.R cCopula.R  
 claytonExpr.R ellipCopula.R empCopula.R empPsi.R acR.R  
 estimation.R evCopula.R evTests.R exchTests.R fgmCopula.R  
 fitCopula.R fitLambda.R fitMvdc.R fixedPar.R frankExpr.R  
 galambosCopula.R galambosExpr-math.R galambosExpr.R  
 ggraph-tools.R pairsRosenblatt.R prob.R gofTrafos.R  
 gofEVTTests.R gofCopula.R graphics.R gumbelCopula.R gumbelExpr.R  
 huslerReissCopula.R huslerReissExpr.R indepCopula.R fhCopula.R  
 lowfhCopula.R upfhCopula.R indepTests.R joeCopula.R K.R  
 logseries.R mvdc.R margCopula.R matrix\_tools.R normalCopula.R  
 obs.R opower.R plackettCopula.R plackettExpr.R moCopula.R  
 rstable1.R safeUroot.R schlatherCopula.R stable.R timing.R  
 tCopula.R tawnCopula.R tawnExpr.R tevCopula.R  
 varianceReduction.R wrapper.R xvCopula.R zzz.R

**Encoding** UTF-8

**URL** <https://copula.r-forge.r-project.org/>,  
<https://r-forge.r-project.org/projects/copula/>,  
<https://CRAN.r-project.org/package=copula>

**BugReports** [https://r-forge.r-project.org/tracker/?func=add&group\\_id=2140&atid=5417](https://r-forge.r-project.org/tracker/?func=add&group_id=2140&atid=5417)

**Repository** <https://r-forge.r-universe.dev>

**RemoteUrl** <https://github.com/r-forge/copula>

**RemoteRef** HEAD

**RemoteSha** 0e796fb0381faa5ce7421fd2423214a2e6a89e2b

## Contents

|                             |    |
|-----------------------------|----|
| copula-package . . . . .    | 5  |
| .pairsCond . . . . .        | 7  |
| absdPsiMC . . . . .         | 8  |
| acopula-class . . . . .     | 10 |
| acR . . . . .               | 13 |
| allComp . . . . .           | 14 |
| An . . . . .                | 15 |
| archmCopula . . . . .       | 17 |
| archmCopula-class . . . . . | 19 |
| assocMeasures . . . . .     | 21 |
| Bernoulli . . . . .         | 22 |
| beta.Blomqvist . . . . .    | 23 |
| cCopula . . . . .           | 25 |
| cloud2-methods . . . . .    | 29 |
| coeffG . . . . .            | 30 |

|                                |     |
|--------------------------------|-----|
| contour-methods . . . . .      | 31  |
| contourplot2-methods . . . . . | 33  |
| copFamilies . . . . .          | 35  |
| Copula . . . . .               | 38  |
| copula-class . . . . .         | 40  |
| corKendall . . . . .           | 41  |
| dDiag . . . . .                | 43  |
| describeCop . . . . .          | 44  |
| dnacopula . . . . .            | 45  |
| ellipCopula . . . . .          | 46  |
| ellipCopula-class . . . . .    | 50  |
| emde . . . . .                 | 51  |
| emle . . . . .                 | 53  |
| empCopula . . . . .            | 56  |
| empCopula-class . . . . .      | 60  |
| enacopula . . . . .            | 60  |
| estim.misc . . . . .           | 63  |
| evCopula . . . . .             | 65  |
| evCopula-class . . . . .       | 67  |
| evTestA . . . . .              | 68  |
| evTestC . . . . .              | 70  |
| evTestK . . . . .              | 71  |
| exchEVTest . . . . .           | 72  |
| exchTest . . . . .             | 74  |
| fgmCopula . . . . .            | 75  |
| fgmCopula-class . . . . .      | 77  |
| fhCopula . . . . .             | 78  |
| fhCopula-class . . . . .       | 79  |
| fitCopula . . . . .            | 79  |
| fitCopula-class . . . . .      | 85  |
| fitLambda . . . . .            | 86  |
| fitMvdc . . . . .              | 88  |
| fixParam . . . . .             | 91  |
| gasoil . . . . .               | 92  |
| generator . . . . .            | 93  |
| getAcop . . . . .              | 94  |
| getIniParam . . . . .          | 96  |
| getTheta . . . . .             | 97  |
| ggraph-tools . . . . .         | 98  |
| gnacopula . . . . .            | 99  |
| gofCopula . . . . .            | 101 |
| gofEVCopula . . . . .          | 106 |
| gofOtherTstat . . . . .        | 108 |
| gofTstat . . . . .             | 109 |
| htrafo . . . . .               | 111 |
| indepCopula . . . . .          | 113 |
| indepCopula-class . . . . .    | 113 |
| indepTest . . . . .            | 114 |

|                                 |     |
|---------------------------------|-----|
| initOpt . . . . .               | 117 |
| interval . . . . .              | 119 |
| interval-class . . . . .        | 120 |
| K . . . . .                     | 121 |
| khoudrajiCopula . . . . .       | 125 |
| khoudrajiCopula-class . . . . . | 129 |
| log1mexp . . . . .              | 130 |
| loss . . . . .                  | 131 |
| margCopula . . . . .            | 132 |
| math-fun . . . . .              | 133 |
| matrix_tools . . . . .          | 134 |
| mixCopula . . . . .             | 136 |
| mixCopula-class . . . . .       | 138 |
| moCopula . . . . .              | 139 |
| moCopula-class . . . . .        | 140 |
| multIndepTest . . . . .         | 141 |
| multSerialIndepTest . . . . .   | 143 |
| Mvdc . . . . .                  | 145 |
| mvdc-class . . . . .            | 147 |
| naCFrail.time . . . . .         | 148 |
| naCopula-class . . . . .        | 149 |
| naPairthetas . . . . .          | 150 |
| nesdepth . . . . .              | 151 |
| onacopula . . . . .             | 152 |
| opower . . . . .                | 154 |
| pairs2 . . . . .                | 156 |
| pairsRosenblatt . . . . .       | 157 |
| persp-methods . . . . .         | 161 |
| plackettCopula . . . . .        | 162 |
| plackettCopula-class . . . . .  | 163 |
| plot-methods . . . . .          | 164 |
| pnacopula . . . . .             | 165 |
| pobs . . . . .                  | 166 |
| polylog . . . . .               | 168 |
| polynEval . . . . .             | 171 |
| printNacopula . . . . .         | 172 |
| prob . . . . .                  | 173 |
| qqplot2 . . . . .               | 174 |
| radSymTest . . . . .            | 175 |
| rdj . . . . .                   | 177 |
| retstable . . . . .             | 178 |
| rF01FrankJoe . . . . .          | 179 |
| rFFrankJoe . . . . .            | 181 |
| rlog . . . . .                  | 182 |
| rnacModel . . . . .             | 183 |
| rnacopula . . . . .             | 184 |
| rnchild . . . . .               | 185 |
| rotCopula . . . . .             | 187 |

|                    |     |
|--------------------|-----|
| RSpobs             | 189 |
| rstable1           | 191 |
| safeUroot          | 193 |
| serialIndepTest    | 195 |
| setTheta           | 197 |
| show-methods       | 199 |
| Sibuya             | 199 |
| SMI.12             | 201 |
| splom2-methods     | 203 |
| Stirling           | 204 |
| tauAMH             | 206 |
| uranium            | 207 |
| varianceReduction  | 208 |
| wireframe2-methods | 211 |
| xvCopula           | 213 |

|              |            |
|--------------|------------|
| <b>Index</b> | <b>215</b> |
|--------------|------------|

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|                |                                                      |
|----------------|------------------------------------------------------|
| copula-package | <i>Multivariate Dependence Modeling with Copulas</i> |
|----------------|------------------------------------------------------|

---

## Description

The **copula** package provides (S4) classes of commonly used elliptical, (nested) Archimedean, extreme value and other copula families; methods for density, distribution, random number generation, and plots.

Fitting copula models and goodness-of-fit tests. Independence and serial (univariate and multivariate) independence tests, and other copula related tests.

## Details

The DESCRIPTION file: This package was not yet installed at build time.

Index: This package was not yet installed at build time.

The **copula** package provides

- Classes (S4) of commonly used copulas including elliptical (normal and t; [ellipCopula](#)), Archimedean (Clayton, Gumbel, Frank, Joe, and Ali-Mikhail-Haq; ; [archmCopula](#) and [acopula](#)), extreme value (Gumbel, Husler-Reiss, Galambos, Tawn, and t-EV; [evCopula](#)), and other families (Plackett and Farlie-Gumbel-Morgenstern).
- Methods for density, distribution, random number generation ([dCopula](#), [pCopula](#) and [rCopula](#)); bivariate dependence measures ([rho](#), [tau](#), etc), perspective and contour plots.
- Functions (and methods) for fitting copula models including variance estimates ([fitCopula](#)).
- Independence tests among random variables and vectors.
- Serial independence tests for univariate and multivariate continuous time series.

- Goodness-of-fit tests for copulas based on multipliers, and the parametric bootstrap, with several transformation options.
- Bivariate and multivariate tests of extreme-value dependence.
- Bivariate tests of exchangeability.

Now with former package **nacopula** for working with nested Archimedean copulas. Specifically,

- it provides procedures for computing function values and cube volumes ([prob](#)),
- characteristics such as Kendall's tau and tail dependence coefficients (via family objects, e.g., [copGumbel](#)),
- efficient sampling algorithms ([rnacopula](#)),
- various estimators and goodness-of-fit tests.
- The package also contains related univariate distributions and special functions such as the Sibuya distribution ([Sibuya](#)), the polylogarithm ([polylog](#)), Stirling and Eulerian numbers ([Eulerian](#)).

Further information is available in the following [vignettes](#):

```
nacopula-pkg  Nested Archimedean Copulas Meet R (../doc/nacopula-pkg.pdf)
Frank-Rmpfr   Numerically Stable Frank via Multiprecision in R (../doc/Frank-Rmpfr)
```

For a list of exported functions, use `help(package = "copula")`.

## References

- Yan, J. (2007) Enjoy the Joy of Copulas: With a Package **copula**. *Journal of Statistical Software* **21**(4), 1–21. <https://www.jstatsoft.org/v21/i04/>.
- Kojadinovic, I. and Yan, J. (2010). Modeling Multivariate Distributions with Continuous Margins Using the copula R Package. *Journal of Statistical Software* **34**(9), 1–20. <https://www.jstatsoft.org/v34/i09/>.
- Hofert, M. and Mächler, M. (2011), Nested Archimedean Copulas Meet R: The nacopula Package., *Journal of Statistical Software* **39**(9), 1–20. <https://www.jstatsoft.org/v39/i09/>.
- Nelsen, R. B. (2006) *An introduction to Copulas*. Springer, New York.

## See Also

The following CRAN packages currently use ('depend on') **copula**: **CoClust**, **copulaedas**, **Depela**, **HAC**, **ipptoolbox**, **vines**.

## Examples

```
## Some of the more important functions (and their examples) are

example(fitCopula)## fitting Copulas
example(fitMvdc)  ## fitting multivariate distributions via Copulas
example(nacopula) ## nested Archimedean Copulas
```

```
## Independence Tests: These also draw a 'Dependogram':
example(indepTest)      ## Testing for Independence
example(serialIndepTest) ## Testing for Serial Independence
```

.pairsCond *Pairs Plot of a cu.u Object (Internal Use)*

### Description

.pairsCond() is an internal function for plotting the pairwise Rosenblatt transforms, i.e., the pairwise conditional distributions, as returned by [pairwiseCcop\(\)](#), via the principal function [pairsRosenblatt\(\)](#). The intention is that [pairsRosenblatt\(\)](#) be called, rather than this auxiliary function.

### Usage

```
.pairsCond(gcu.u, panel = points, collist,
  col = par("col"), bg = par("bg"), labels, ...,
  text.panel = textPanel, label.pos = 0.5,
  cex.labels = NULL, font.labels = 1, gap = 0,
  axes = TRUE, panel.border = TRUE, key = TRUE,
  keyOpt = list(space= 2.5, width= 1.5, axis= TRUE,
    rug.at= numeric(), title= NULL, line= 5),
  main = NULL, main.centered = FALSE,
  line.main = if(is.list(main)) 5/4*par("cex.main")* rev(seq_along(main)) else 2,
  sub = NULL, sub.centered = FALSE, line.sub = 4)
```

### Arguments

- gcu.u (n,d,d)-array of pairwise Rosenblatt-transformed u's as returned by [pairwiseCcop\(\)](#).
- panel panel function, as for [pairs\(\)](#).
- collist list of colors and information as returned by [pairsCollist\(\)](#).
- col *instead of collist*, specifying the points' color.
- bg *instead of collist*, specifying the constant background color.
- labels [pairs\(\)](#) argument; can be missing (in which case a suitable default is chosen or can be "none" [or something else])
- ... further arguments, as for [pairs](#). These are passed to [panel\(\)](#), and [axis](#), may also contain `font.main`, `cex.main`, and `adj`, for title adjustments; further, `oma` for modifying the default `par("oma")`.
- text.panel, label.pos, cex.labels, font.labels, gap see [pairs\(\)](#).
- axes logical indicating whether axes are drawn.
- panel.border logical indicating whether a border is drawn around the pairs (to mimic the behavior of [image\(\)](#)).

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|---------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| key           | logical indicating whether a color key is drawn.                                                                                                                                                                                                                                                                                                                                                                                                  |
| keyOpt        | a <a href="#">list</a> of options for the color key;<br>space: white space in height of characters in inch to specify the the distance of the key to the pairs plot.<br>width: key width in height of characters in inch.<br>axis: logical indicating whether an axis for the color key is drawn.<br>rug.at: values where rugs are plotted at the key.<br>title: key title.<br>line: key placement (horizontal distance from color key in lines). |
| main          | title                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| main.centered | logical indicating if the title should be centered or not; the default FALSE centers it according to the pairs plot, not the whole plotting region.                                                                                                                                                                                                                                                                                               |
| line.main     | title placement (vertical distance from pairs plot in lines).                                                                                                                                                                                                                                                                                                                                                                                     |
| sub           | sub-title                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| sub.centered  | logical indicating if the sub-title should be centered or not; see main.centered.                                                                                                                                                                                                                                                                                                                                                                 |
| line.sub      | sub-title placement, see line.main.                                                                                                                                                                                                                                                                                                                                                                                                               |

**Note**

based on `pairs.default()` and `filled.contour()` from R-2.14.1 - used in Hofert and Maechler (2013)

**Author(s)**

Marius Hofert and Martin Maechler

**See Also**

[pairsRosenblatt\(\)](#), the prinicipal function, calling `.pairsCond()`.

---

absdPsiMC

*Absolute Value of Generator Derivatives via Monte Carlo*


---

**Description**

Computes the absolute values of the  $d$ th generator derivative  $\psi^{(d)}$  via Monte Carlo simulation.

**Usage**

```
absdPsiMC(t, family, theta, degree = 1, n.MC,
          method = c("log", "direct", "pois.direct", "pois"),
          log = FALSE, is.log.t = FALSE)
```



**Arguments**

|          |                                                                                                                                                                                                                                                                                                                                                                                                                             |
|----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| t        | numeric vector of evaluation points.                                                                                                                                                                                                                                                                                                                                                                                        |
| family   | Archimedean family (name or object).                                                                                                                                                                                                                                                                                                                                                                                        |
| theta    | parameter value.                                                                                                                                                                                                                                                                                                                                                                                                            |
| degree   | order $d$ of the derivative.                                                                                                                                                                                                                                                                                                                                                                                                |
| n.MC     | Monte Carlo sample size.                                                                                                                                                                                                                                                                                                                                                                                                    |
| method   | different methods:<br>"log": evaluates the logarithm of the sum involved in the Monte Carlo approximation in a numerically stable way;<br>"direct": directly evaluates the sum;<br>"pois.direct": interprets the sum in terms of the density of a Poisson distribution and evaluates this density directly;<br>"pois": as for method="pois" but evaluates the logarithm of the Poisson density in a numerically stable way. |
| log      | if TRUE the logarithm of absdPsi is returned.                                                                                                                                                                                                                                                                                                                                                                               |
| is.log.t | if TRUE the argument t contains the logarithm of the "mathematical" $t$ , i.e., conceptually, $\text{psi}(t, *) = \text{psi}(\log(t), *, \text{is.log.t}=\text{TRUE})$ , where the latter may potentially be numerically accurate, e.g., for $t = 10^{500}$ , where as the former would just return $\text{psi}(\text{Inf}, *) = 0$ .                                                                                       |

**Details**

The absolute value of the  $d$ th derivative of the Laplace-Stieltjes transform  $\psi = \mathcal{L}[F]$  can be approximated via

$$(-1)^d \psi^{(d)}(t) = \int_0^\infty x^d \exp(-tx) dF(x) \approx \frac{1}{N} \sum_{k=1}^N V_k^d \exp(-V_k t), \quad t > 0,$$

where  $V_k \sim F$ ,  $k \in \{1, \dots, N\}$ . This approximation is used where  $d = \text{degree}$  and  $N = \text{n.MC}$ . Note that this is comparably fast even if t contains many evaluation points, since the random variates  $V_k \sim F$ ,  $k \in \{1, \dots, N\}$  only have to be generated once, not depending on t.

**Value**

numeric vector of the same length as t containing the absolute values of the generator derivatives.

**References**

Hofert, M., Mächler, M., and McNeil, A. J. (2013). Archimedean Copulas in High Dimensions: Estimators and Numerical Challenges Motivated by Financial Applications. *Journal de la Société Française de Statistique* **154**(1), 25–63.

**See Also**

[acopula-families](#).

## Examples

```
t <- c(0:100,Inf)
set.seed(1)
(ps <- absdPsiMC(t, family="Gumbel", theta=2, degree=10, n.MC=10000, log=TRUE))
## Note: The absolute value of the derivative at 0 should be Inf for
## Gumbel, however, it is always finite for the Monte Carlo approximation
set.seed(1)
ps2 <- absdPsiMC(log(t), family="Gumbel", theta=2, degree=10,
                 n.MC=10000, log=TRUE, is.log.t = TRUE)
stopifnot(all.equal(ps[-1], ps2[-1], tolerance=1e-14))
## Now is there an advantage of using "is.log.t" ?
sapply(eval(formals(absdPsiMC)$method), function(MM)
       absdPsiMC(780, family="Gumbel", method = MM,
                 theta=2, degree=10, n.MC=10000, log=TRUE, is.log.t = TRUE))
## not really better, yet...
```

---

acopula-class

*Class "acopula" of Archimedean Copula Families*

---

## Description

This class "acopula" of Archimedean Copula Families is mainly used for providing objects of known Archimedean families with all related functions.

## Objects from the Class

Objects can be created by calls of the form `new("acopula", ...)`. For several well-known Archimedean copula families, the package **copula** already provides such family objects.

## Slots

**name:** A string (class "character") describing the copula family, for example, "AMH" (or simply "A"), "Clayton" ("C"), "Frank" ("F"), "Gumbel" ("G"), or "Joe" ("J").

**theta:** Parameter value, a **numeric**, where NA means "unspecified".

**psi, iPsi:** The (Archimedean) generator  $\psi$  (with  $\psi(t)=\exp(-t)$  being the generator of the independence copula) and its inverse (**function**). **iPsi** has an optional argument **log** which, if TRUE returns the logarithm of inverse of the generator.

**absdPsi:** A **function** which computes the absolute value of the derivative of the generator  $\psi$  for the given parameter **theta** and of the given degree (defaults to 1). Note that there is no informational loss by computing the absolute value since the derivatives alternate in sign (the generator derivative is simply  $(-1)^{\text{degree}} \cdot \text{absdPsi}$ ). The number **n.MC** denotes the sample size for a Monte Carlo evaluation approach. If **n.MC** is zero (the default), the generator derivatives are evaluated with their exact formulas. The optional parameter **log** (defaults to FALSE) indicates whether or not the logarithmic value is returned.

**absdiPsi:** a **function** computing the absolute value of the derivative of the generator inverse (**iPsi()**) for the given parameter **theta**. The optional parameter **log** (defaults to FALSE) indicates whether the logarithm of the absolute value of the first derivative of **iPsi()** is returned.

- dDiag:** a **function** computing the density of the diagonal of the Archimedean copula at  $u$  with parameter  $\theta$ . The parameter `log` is as described before.
- dacopula:** a **function** computing the density of the Archimedean copula at  $u$  with parameter  $\theta$ . The meanings of the parameters `n.MC` and `log` are as described before.
- score:** a **function** computing the *derivative* of the density with respect to the parameter  $\theta$ .
- uscore:** a **function** computing the *derivative* of the density with respect to the each of the arguments.
- paraInterval:** Either `NULL` or an object of class `"interval"`, which is typically obtained from a call such as `interval("[a,b]")`.
- paraConstr:** A function of  $\theta$  returning `TRUE` if and only if  $\theta$  is a valid parameter value. Note that `paraConstr` is built automatically from the `interval`, whenever the `paraInterval` slot is valid. `"interval"`.
- nestConstr:** A **function**, which returns `TRUE` if and only if the two provided parameters `theta0` and `theta1` satisfy the sufficient nesting condition for this family.
- V0:** A **function** which samples  $n$  random variates from the distribution  $F$  with Laplace-Stieltjes transform  $\psi$  and parameter  $\theta$ .
- dV0:** A **function** which computes either the probability mass function or the probability density function (depending on the Archimedean family) of the distribution function whose Laplace-Stieltjes transform equals the generator  $\psi$  at the argument  $x$  (possibly a vector) for the given parameter  $\theta$ . An optional argument `log` indicates whether the logarithm of the mass or density is computed (defaults to `FALSE`).
- V01:** A **function** which gets a vector of realizations of  $V0$ , two parameters `theta0` and `theta1` which satisfy the sufficient nesting condition, and which returns a vector of the same length as  $V0$  with random variates from the distribution function  $F_{01}$  with Laplace-Stieltjes transform  $\psi_{01}$  (see `dV01`) and parameters  $\theta_0 = \theta_0$ ,  $\theta_1 = \theta_1$ .
- dV01:** Similar to `dV0` with the difference being that this **function** computes the probability mass or density function for the Laplace-Stieltjes transform

$$\psi_{01}(t; V_0) = \exp(-V_0 \psi_0^{-1}(\psi_1(t))),$$

corresponding to the distribution function  $F_{01}$ .

Arguments are the evaluation point(s)  $x$ , the value(s)  $V_0$ , and the parameters `theta0` and `theta1`. As for `dV0`, the optional argument `log` can be specified (defaults to `FALSE`). Note that if  $x$  is a vector,  $V_0$  must either have length one (in which case  $V_0$  is the same for every component of  $x$ ) or  $V_0$  must be of the same length as  $x$  (in which case the components of  $V_0$  correspond to the ones of  $x$ ).

- tau, iTau:** Compute Kendall's tau of the bivariate Archimedean copula with generator  $\psi$  as a **function** of  $\theta$ , respectively,  $\theta$  as a function of Kendall's tau.
- lambdaL, lambdaU, lambdaLInv, lambdaUInv:** Compute the lower (upper) tail-dependence coefficient of the bivariate Archimedean copula with generator  $\psi$  as a **function** of  $\theta$ , respectively,  $\theta$  as a function of the lower (upper) tail-dependence coefficient.

For more details about Archimedean families, corresponding distributions and properties, see the references.

## Methods

**initialize** signature(.Object = "acopula"): is used to automatically construct the function slot paraConstr, when the paraInterval is provided (typically via [interval\(\)](#)).

**show** signature("acopula"): compact overview of the copula.

## References

See those of the families, for example, [copGumbel](#).

## See Also

Specific provided copula family objects, for example, [copAMH](#), [copClayton](#), [copFrank](#), [copGumbel](#), [copJoe](#).

To access these, you may also use [getAcop](#).

A *nested* Archimedean copula *without* child copulas (see class "[nacopula](#)") is a proper Archimedean copula, and hence, [onacopula\(\)](#) can be used to construct a specific parametrized Archimedean copula; see the example below.

Alternatively, [setTheta](#) also returns such a (parametrized) Archimedean copula.

## Examples

```
## acopula class information
showClass("acopula")

## Information and structure of Clayton copulas
copClayton
str(copClayton)

## What are admissible parameters for Clayton copulas?
copClayton@paraInterval

## A Clayton "acopula" with Kendall's tau = 0.8 :
(cCl.2 <- setTheta(copClayton, iTau(copClayton, 0.8)))

## Can two Clayton copulas with parameters theta0 and theta1 be nested?
## Case 1: theta0 = 3, theta1 = 2
copClayton@nestConstr(theta0 = 3, theta1 = 2)
## -> FALSE as the sufficient nesting criterion is not fulfilled
## Case 2: theta0 = 2, theta1 = 3
copClayton@nestConstr(theta0 = 2, theta1 = 3) # TRUE

## For more examples, see help("acopula-families")
```

**Description**

pacR() computes the distribution function  $F_R$  of the radial part of an Archimedean copula, given by

$$F_R(x) = 1 - \sum_{k=0}^{d-1} \frac{(-x)^k \psi^{(k)}(x)}{k!}, \quad x \in [0, \infty);$$

The formula (in a slightly more general form) is given by McNeil and G. Nešlehová (2009).

qacR() computes the quantile function of  $F_R$ .

**Usage**

```
pacR(x, family, theta, d, lower.tail = TRUE, log.p = FALSE, ...)
qacR(p, family, theta, d, log.p = FALSE, interval,
     tol = .Machine$double.eps^0.25, maxiter = 1000, ...)
```

**Arguments**

|            |                                                                                   |
|------------|-----------------------------------------------------------------------------------|
| x          | numeric vector of nonnegative evaluation points for $F_R$ .                       |
| p          | numeric vector of evaluation points of the quantile function.                     |
| family     | Archimedean family.                                                               |
| theta      | parameter <i>theta</i> .                                                          |
| d          | dimension <i>d</i> .                                                              |
| lower.tail | <b>logical</b> ; if TRUE, probabilities are $P[X \leq x]$ otherwise, $P[X > x]$ . |
| log.p      | <b>logical</b> ; if TRUE, probabilities <i>p</i> are given as $\log p$ .          |
| interval   | root-search interval.                                                             |
| tol        | see <code>uniroot()</code> .                                                      |
| maxiter    | see <code>uniroot()</code> .                                                      |
| ...        | additional arguments passed to the procedure for computing derivatives.           |

**Value**

The distribution function of the radial part evaluated at *x*, or its inverse, the quantile at *p*.

**References**

McNeil, A. J., G. Nešlehová, J. (2009). Multivariate Archimedean copulas, *d*-monotone functions and  $l_1$ -norm symmetric distributions. *The Annals of Statistics* **37**(5b), 3059–3097.

**Examples**

```
## setup
family <- "Gumbel"
tau <- 0.5
m <- 256
dmax <- 20
x <- seq(0, 20, length.out=m)

## compute and plot pacR() for various d's
y <- vapply(1:dmax, function(d)
  pacR(x, family=family, theta=iTau(archmCopula(family), tau), d=d),
  rep(NA_real_, m))
plot(x, y[,1], type="l", ylim=c(0,1),
  xlab = quote(italic(x)), ylab = quote(F[R](x)),
  main = substitute(italic(F[R](x))~ "for" ~ d==1:.D, list(.D = dmax)))
for(k in 2:dmax) lines(x, y[,k])
```

---

allComp

*All Components of a (Inner or Outer) Nested Archimedean Copula*


---

**Description**

Given the nested Archimedean copula  $x$ , return an integer vector of the *indices* of all components of the corresponding [outer\\_nacopula](#) which are components of  $x$ , either direct components or components of possible child copulas. This is typically only used by programmers investigating the exact nesting structure.

For an [outer\\_nacopula](#) object  $x$ , `allComp(x)` must be the same as `1:dim(x)`, whereas its “inner” component copulas will each contain a *subset* of those indices only.

**Usage**

```
allComp(x)
```

**Arguments**

$x$  an R object inheriting from class [nacopula](#).

**Value**

An [integer](#) vector of indices  $j$  of all components  $u_j$  as described in the description above.

**Examples**

```
C3 <- onacopula("AMH", C(0.7135, 1, C(0.943, 2:3)))
allComp(C3) # components are 1:3
allComp(C3@childCops[[1]]) # for the child, only (2, 3)
```

---

An *Nonparametric Rank-based Estimators of the Pickands Dependence Function*

---

### Description

Bivariate and multivariate versions of the nonparametric rank-based estimators of the Pickands dependence function  $A$ , studied in Genest and Segers (2009) and Gudendorf and Segers (2011).

### Usage

```
An.biv(x, w, estimator = c("CFG", "Pickands"), corrected = TRUE,
      ties.method = eval(formals(rank)$ties.method))
An(x, w, ties.method = eval(formals(rank)$ties.method))
```

### Arguments

|             |                                                                                                                                                                                                                                                                                                                                                        |
|-------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x           | a data matrix that will be transformed to pseudo-observations. If <code>An.biv</code> is called, x has to have two columns.                                                                                                                                                                                                                            |
| w           | if <code>An.biv</code> is called, a vector of points in $[0,1]$ where to evaluate the estimated bivariate Pickands dependence function. If the multivariate estimator <code>An</code> is used instead, w needs to be a matrix with the same number of columns as x whose lines are elements of the multivariate unit simplex (see the last reference). |
| estimator   | specifies which nonparametric rank-based estimator of the unknown Pickands dependence function to use in the bivariate case; can be either "CFG" (Capéraà-Fougères-Genest) or "Pickands".                                                                                                                                                              |
| corrected   | TRUE means that the bivariate estimators will be corrected to ensure that their value at 0 and 1 is 1.                                                                                                                                                                                                                                                 |
| ties.method | <a href="#">character</a> string specifying how ranks should be computed if there are ties in any of the coordinate samples of x; passed to <a href="#">pobs</a> .                                                                                                                                                                                     |

### Details

More details can be found in the references.

### Value

`An.biv()` returns a vector containing the values of the estimated Pickands dependence function at the points in w (and is the same as former `Anfun()`).

The function `An` computes simultaneously the three corrected multivariate estimators studied in Gudendorf and Segers (2011) at the points in w and returns a list whose components are

|     |                                                          |
|-----|----------------------------------------------------------|
| P   | values of the Pickands estimator at the points in w.     |
| CFG | values of the CFG estimator at the points in w.          |
| HT  | values of the Hall-Tajvidi estimator at the points in w. |

## References

C. Genest and J. Segers (2009). Rank-based inference for bivariate extreme-value copulas. *Annals of Statistics* **37**, 2990–3022.

G. Gudendorf and J. Segers (2011). Nonparametric estimation of multivariate extreme-value copulas. *Journal of Statistical Planning and Inference* **142**, 3073–3085.

## See Also

[evCopula](#), [A](#), and [evTestA](#). Further, [evTestC](#), [evTestK](#), [exchEVTTest](#), and [gofEVCopula](#).

## Examples

```
## True Pickands dependence functions
curve(A(gumbelCopula(4  ), x), 0, 1)
curve(A(gumbelCopula(2  ), x), add=TRUE, col=2)
curve(A(gumbelCopula(1.33), x), add=TRUE, col=3)

## CFG estimator
curve(An.biv(rCopula(1000, gumbelCopula(4  )), x), lty=2, add=TRUE)
curve(An.biv(rCopula(1000, gumbelCopula(2  )), x), lty=2, add=TRUE, col=2)
curve(An.biv(rCopula(1000, gumbelCopula(1.33)), x), lty=2, add=TRUE, col=3)

## Pickands estimator
curve(An.biv(rCopula(1000, gumbelCopula(4  )), x, estimator="Pickands"),
      lty=3, add=TRUE)
curve(An.biv(rCopula(1000, gumbelCopula(2  )), x, estimator="Pickands"),
      lty=3, add=TRUE, col=2)
curve(An.biv(rCopula(1000, gumbelCopula(1.33)), x, estimator="Pickands"),
      lty=3, add=TRUE, col=3)

legend("bottomleft", paste0("Gumbel(", format(c(4, 2, 1.33)),")"),
      lwd=1, col=1:3, bty="n")
legend("bottomright", c("true", "CFG est.", "Pickands est."), lty=1:3, bty="n")

## Relationship between An.biv and An
u <- c(runif(100),0,1) # include 0 and 1
x <- rCopula(1000, gumbelCopula(4))
r <- An(x, cbind(1-u, u))
all.equal(r$CFG, An.biv(x, u))
all.equal(r$P, An.biv(x, u, estimator="Pickands"))

## A trivariate example
x <- rCopula(1000, gumbelCopula(4, dim = 3))
u <- matrix(runif(300), 100, 3)
w <- u / apply(u, 1, sum)
r <- An(x, w)

## Endpoint corrections are applied
An(x, cbind(1, 0, 0))
An(x, cbind(0, 1, 0))
An(x, cbind(0, 0, 1))
```



**Description**

Constructs an Archimedean copula class object with its corresponding parameter and dimension.

**Usage**

```
archmCopula(family, param = NA_real_, dim = 2, ...)

claytonCopula(param = NA_real_, dim = 2,
  use.indepC = c("message", "TRUE", "FALSE"))
frankCopula(param = NA_real_, dim = 2,
  use.indepC = c("message", "TRUE", "FALSE"))
gumbelCopula(param = NA_real_, dim = 2,
  use.indepC = c("message", "TRUE", "FALSE"))
amhCopula(param = NA_real_, dim = 2,
  use.indepC = c("message", "TRUE", "FALSE"))
joeCopula(param = NA_real_, dim = 2,
  use.indepC = c("message", "TRUE", "FALSE"))
```

**Arguments**

|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| family     | a character string specifying the family of an Archimedean copula. Currently supported families are "clayton", "frank", "amh", "gumbel", and "joe".                                                                                                                                                                                                                                                                                                                                                                                                                    |
| param      | number ( <b>numeric</b> ) specifying the copula parameter.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
| dim        | the dimension of the copula.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| ...        | further arguments, passed to the individual creator functions (claytonCopula(), etc).                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| use.indepC | a string specifying if the independence copula <a href="#">indepCopula</a> , should be returned in the case where the parameter $\theta$ , param, is at the boundary or limit case where the corresponding Archimedean copula is the independence copula. The default does return indepCopula() with a message, using "TRUE" does it without a message. This makes the resulting object more useful typically, but does not return a formal Archimedean copula of the desired family, something needed e.g., for fitting purposes, where you'd use use.indepC="FALSE". |

**Details**

archmCopula() is a wrapper for claytonCopula(), frankCopula(), gumbelCopula(), amhCopula() and joeCopula.

For the mathematical definitions of the respective Archimedean families, see [copClayton](#).

For  $d = 2$ , i.e.  $\text{dim} = 2$ , the AMH, Clayton and Frank copulas allow to model negative Kendall's tau (**tau**) behavior via negative  $\theta$ , for AMH and Clayton  $-1 \leq \theta$ , and for Frank  $-\infty < \theta$ . The respective boundary cases are

**AMH:**  $\tau(\theta = -1) = -0.1817258$ ,

**Clayton:**  $\tau(\theta = -1) = -1$ ,

**Frank:**  $\tau(\theta = -Inf) = -1$  (as limit).

For the Ali-Mikhail-Haq copula family ("**amhCopula**"), only the bivariate case is available; however **copAMH** has no such limitation.

Note that in all cases except for Frank and AMH, and  $d = 2$  and  $\theta < 0$ , the densities (**dCopula**() methods) are evaluated via the **dacopula** slot of the corresponding **acopula**-classed Archimedean copulas, implemented in a numeric stable way without any restriction on the dimension  $d$ .

Similarly, the (cumulative) distribution function ("the copula"  $C()$ ) is evaluated via the corresponding **acopula**-classed Archimedean copula's functions in the **psi** and **iPsi** slots.

## Value

An Archimedean copula object of class "**claytonCopula**", "**frankCopula**", "**gumbelCopula**", "**amhCopula**", or "**joeCopula**".

## References

R.B. Nelsen (2006), *An introduction to Copulas*, Springer, New York.

## See Also

**acopula**-classed Archimedean copulas, such as **copClayton**, **copGumbel**, etc, notably for mathematical definitions including the meaning of **param**.

**fitCopula** for fitting such copulas to data.

**ellipCopula**, **evCopula**.

## Examples

```
clayton.cop <- claytonCopula(2, dim = 3)
## scatterplot3d(rCopula(1000, clayton.cop))

## negative param (= theta) is allowed for dim = 2 :
tau(claytonCopula(-0.5)) ## = -1/3
tauClayton <- Vectorize(function(theta) tau(claytonCopula(theta, dim=2)))
plot(tauClayton, -1, 10, xlab=quote(theta), ylim = c(-1,1), n=1025)
abline(h=-1:1,v=0, col="#11111150", lty=2); axis(1, at=-1)

tauFrank <- Vectorize(function(theta) tau(frankCopula(theta, dim=2)))
plot(tauFrank, -40, 50, xlab=quote(theta), ylim = c(-1,1), n=1025)
abline(h=-1:1,v=0, col="#11111150", lty=2)

## tauAMH() is function in our package
iTau(amhCopula(), -1) # -1 with a range warning
```

```

iTau(amhCopula(), (5 - 8*log(2)) / 3) # -1 with a range warning

ic <- frankCopula(0) # independence copula (with a "message")
stopifnot(identical(ic,
  frankCopula(0, use.indepC = "TRUE")))# indep.copula withOUT message
(fc <- frankCopula(0, use.indepC = "FALSE"))
## a Frank copula which corresponds to the indep.copula (but is not)

frankCopula(dim = 3)# with NA parameters
frank.cop <- frankCopula(3)# dim=2
persp(frank.cop, dCopula)

gumbel.cop <- archmCopula("gumbel", 5)
stopifnot(identical(gumbel.cop, gumbelCopula(5)))
contour(gumbel.cop, dCopula)

amh.cop <- amhCopula(0.5)
u. <- as.matrix(expand.grid(u=(0:10)/10, v=(0:10)/10, KEEP.OUT.ATTRS=FALSE))
du <- dCopula(u., amh.cop)
stopifnot(is.finite(du) | apply(u. == 0, 1,any)| apply(u. == 1, 1,any))

## A 7-dim Frank copula
frank.cop <- frankCopula(3, dim = 7)
x <- rCopula(5, frank.cop)
## dCopula now *does* work:
dCopula(x, frank.cop)

## A 7-dim Gumbel copula
gumbelC.7 <- gumbelCopula(2, dim = 7)
dCopula(x, gumbelC.7)

## A 12-dim Joe copula
joe.cop <- joeCopula(iTau(joeCopula(), 0.5), dim = 12)
x <- rCopula(5, joe.cop)
dCopula(x, joe.cop)

```

---

archmCopula-class      *Class "archmCopula"*

---

## Description

Archimedean copula class.

## Objects from the Class

Created by calls of the form `new("archmCopula", ...)` or rather typically by `archmCopula()`. Implemented families are Clayton, Gumbel, Frank, Joe, and Ali-Mikhail-Haq.

**Slots**

**exprdist:** Object of class "expression": expressions of the cdf and pdf of the copula. These expressions are used in function [pCopula](#) and [dCopula](#).

**dimension, parameters, etc:** all inherited from the super class [copula](#).

**Methods**

**dCopula** signature(copula = "claytonCopula"): ...

**pCopula** signature(copula = "claytonCopula"): ...

**rCopula** signature(copula = "claytonCopula"): ...

**dCopula** signature(copula = "frankCopula"): ...

**pCopula** signature(copula = "frankCopula"): ...

**rCopula** signature(copula = "frankCopula"): ...

**dCopula** signature(copula = "gumbelCopula"): ...

**pCopula** signature(copula = "gumbelCopula"): ...

**rCopula** signature(copula = "gumbelCopula"): ...

**dCopula** signature(copula = "amhCopula"): ...

**pCopula** signature(copula = "amhCopula"): ...

**rCopula** signature(copula = "amhCopula"): ...

**dCopula** signature(copula = "joeCopula"): ...

**pCopula** signature(copula = "joeCopula"): ...

**rCopula** signature(copula = "joeCopula"): ...

**Extends**

Class "archmCopula" extends class "[copula](#)" directly. Class "claytonCopula", "frankCopula", "gumbelCopula", "amhCopula" and "joeCopula" extends class "archmCopula" directly.

**Note**

"gumbelCopula" is also of class "[evCopula](#)".

**See Also**

[archmCopula](#), for constructing such copula objects; [copula-class](#).

## Description

These functions compute Kendall's tau, Spearman's rho, and the tail dependence index for *bivariate* copulas. `iTau` and `iRho`, sometimes called “calibration” functions are the inverses: they determine (“calibrate”) the copula parameter (which must be one-dimensional!) given the value of Kendall's tau or Spearman's rho.

## Usage

```
tau (copula, ...)
rho (copula, ...)
lambda(copula, ...)
iTau (copula, tau, ...)
iRho (copula, rho, ...)
```

## Arguments

|                     |                                                                                                                                                                                             |
|---------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>copula</code> | an R object of class “ <code>copula</code> ” (or also “ <code>acopula</code> ” or “ <code>nacopula</code> ”; note however that some methods may not be available for some copula families). |
| <code>tau</code>    | a numerical value of Kendall's tau in $[-1, 1]$ .                                                                                                                                           |
| <code>rho</code>    | a numerical value of Spearman's rho in $[-1, 1]$ .                                                                                                                                          |
| <code>...</code>    | currently nothing.                                                                                                                                                                          |

## Details

The calibration functions `iTau()` and `iRho()` in fact return a moment estimate of the parameter for one-parameter copulas.

When there are no closed-form expressions for Kendall's tau or Spearman's rho, the calibration functions use numerical approximation techniques (see the last reference). For closed-form expressions, see Frees and Valdez (1998). For the t copula, the calibration function based on Spearman's rho uses the corresponding expression for the normal copula as an approximation.

## References

- E.W. Frees and E.A. Valdez (1998) Understanding relationships using copulas. *North American Actuarial Journal* **2**, 1–25.
- Iwan Kojadinovic and Jun Yan (2010) Comparison of three semiparametric methods for estimating dependence parameters in copula models. *Insurance: Mathematics and Economics* **47**, 52–63. [doi:10.1016/j.insmatheco.2010.03.008](https://doi.org/10.1016/j.insmatheco.2010.03.008)

## See Also

The `acopula` class objects have slots, `tau`, `lambdaL`, and `lambdaU` providing functions for `tau()`, and the two tail indices `lambda()`, and slot `iTau` for `iTau()`, see the examples and `copGumbel`, etc.

**Examples**

```

gumbel.cop <- gumbelCopula(3)
tau(gumbel.cop)
rho(gumbel.cop)
lambda(gumbel.cop)
iTau(joeCopula(), 0.5)
stopifnot(all.equal(tau(gumbel.cop), copGumbel@tau(3)),

          all.equal(lambda(gumbel.cop),
                    c(copGumbel@lambdaL(3), copGumbel@lambdaU(3)),
                    check.attributes=FALSE),

          all.equal(iTau(gumbel.cop, 0.681),
                    copGumbel@iTau(0.681))

)

## let us compute the sample versions
x <- rCopula(200, gumbel.cop)
cor(x, method = "kendall")
cor(x, method = "spearman")
## compare with the true parameter value 3
iTau(gumbel.cop, cor(x, method="kendall" ) [1,2])
iRho(gumbel.cop, cor(x, method="spearman") [1,2])

```

Bernoulli

*Compute Bernoulli Numbers***Description**

Compute the  $n$ th Bernoulli number, or generate all Bernoulli numbers up to the  $n$ th, using diverse methods, that is, algorithms.

**NOTE** the current default methods will be changed – to get better accuracy!

**Usage**

```

Bernoulli      (n, method = c("sumBin", "sumRamanujan", "asymptotic"),
               verbose = FALSE)
Bernoulli.all(n, method = c("A-T", "sumBin", "sumRamanujan", "asymptotic"),
              precBits = NULL, verbose = getOption("verbose"))

```

**Arguments**

**n** positive integer, indicating the index of the largest (and last) of the Bernoulli numbers needed.

**method** character string, specifying which method should be applied. The default for `Bernoulli.all()`, "A-T" stands for the Akiyama-Tanigawa algorithm which is nice and simple but has bad numerical properties. It can however work with high precision "mpfr"-numbers, see `precBits`. "sumRamanujan" is somewhat more efficient but not yet implemented.

|          |                                                                                                                                                                                  |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| precBits | currently only for method = "A-T" – NULL or a positive integer indicating the precision of the initial numbers in bits, using package <b>Rmpfr</b> 's multiprecision arithmetic. |
| verbose  | (for "A-T":) logical indicating if the intermediate results of the algorithm should be printed.                                                                                  |

### Value

Bernoulli(): a number

Bernoulli.all(): a numeric vector of length n, containing B(n)

### References

Kaneko, Masanobu (2000) The Akiyama-Tanigawa algorithm for Bernoulli numbers; Journal of Integer Sequences **3**, article 00.2.9

### See Also

[Eulerian](#), [Stirling1](#), etc.

### Examples

```
## The example for the paper
MASS::fractions(Bernoulli.all(8, verbose=TRUE))

B10 <- Bernoulli.all(10)
MASS::fractions(B10)

system.time(B50 <- Bernoulli.all(50))# {does not cache} -- still "no time"
system.time(B100 <- Bernoulli.all(100))# still less than a milli second

## Using Bernoulli() is not much slower, but hopefully *more* accurate!
## Check first - TODO
system.time(B.1c <- Bernoulli(100))# caches ..
system.time(B1c. <- Bernoulli(100))# ==> now much faster
stopifnot(identical(B.1c, B1c.))

if(FALSE)## reset the cache:
assign("Bern.tab", list(), envir = copula:::nacopEnv)

## More experiments in the source of the copula package ../tests/Stirling-etc.R
```

**Description**

Compute the population (`beta.()`) and sample (`betan()`) version of Blomqvist's beta for an Archimedean copula.

See the reference below for definitions and formulas.

**Usage**

```
beta.(cop, theta, d, scaling=FALSE)
betan(u, scaling=FALSE)
```

**Arguments**

|                      |                                                                                                                       |
|----------------------|-----------------------------------------------------------------------------------------------------------------------|
| <code>cop</code>     | an Archimedean copula (of dimension $d$ ) to be estimated.                                                            |
| <code>theta</code>   | copula parameter.                                                                                                     |
| <code>d</code>       | dimension.                                                                                                            |
| <code>scaling</code> | logical, if true, the factors $2^{(d-1)}/(2^{(d-1)}-1)$ and $2^{(1-d)}$ in Blomqvist's beta are omitted.              |
| <code>u</code>       | For <code>betan</code> : $(n \times d)$ -matrix of $d$ -dimensional observations distributed according to the copula. |

**Value**

`beta.:` a number, being the population version of Blomqvist's beta for the corresponding Archimedean copula;

`betan:` a number, being the sample version of Blomqvist's beta for the given data.

**References**

Schmid and Schmidt (2007), Nonparametric inference on multivariate versions of Blomqvist's beta and related measures of tail dependence, *Metrika* **66**, 323–354.

**See Also**

[acopula](#)

**Examples**

```
beta.(copGumbel, 2.5, d = 5)

d.set <- c(2:6, 8, 10, 15, 20, 30)
cols <- adjustcolor(colorRampPalette(c("red", "orange", "blue")),
                    space = "Lab")(length(d.set)), 0.8)

## AMH:
for(i in seq_along(d.set))
  curve(Vectorize(beta., "theta")(copAMH, x, d = d.set[i]), 0, .999999,
        main = "Blomqvist's beta(.) for AMH",
        xlab = quote(theta), ylab = quote(beta(theta, AMH)),
        add = (i > 1), lwd=2, col=cols[i])
```



```

mtext("NB: d=2 and d=3 are the same")
legend("topleft", paste("d =",d.set), bty="n", lwd=2, col=cols)

## Gumbel:
for(i in seq_along(d.set))
  curve(Vectorize(beta.,"theta")(copGumbel, x, d = d.set[i]), 1, 10,
        main = "Blomqvist's beta(.) for Gumbel",
        xlab = quote(theta), ylab = quote(beta(theta, Gumbel)),
        add=(i > 1), lwd=2, col=cols[i])
legend("bottomright", paste("d =",d.set), bty="n", lwd=2, col=cols)

## Clayton:
for(i in seq_along(d.set))
  curve(Vectorize(beta.,"theta")(copClayton, x, d = d.set[i]), 1e-5, 10,
        main = "Blomqvist's beta(.) for Clayton",
        xlab = quote(theta), ylab = quote(beta(theta, Gumbel)),
        add=(i > 1), lwd=2, col=cols[i])
legend("bottomright", paste("d =",d.set), bty="n", lwd=2, col=cols)

## Joe:
for(i in seq_along(d.set))
  curve(Vectorize(beta.,"theta")(copJoe, x, d = d.set[i]), 1, 10,
        main = "Blomqvist's beta(.) for Joe",
        xlab = quote(theta), ylab = quote(beta(theta, Gumbel)),
        add=(i > 1), lwd=2, col=cols[i])
legend("bottomright", paste("d =",d.set), bty="n", lwd=2, col=cols)

## Frank:
for(i in seq_along(d.set))
  curve(Vectorize(beta.,"theta")(copFrank, x, d = d.set[i]), 1e-5, 50,
        main = "Blomqvist's beta(.) for Frank",
        xlab = quote(theta), ylab = quote(beta(theta, Gumbel)),
        add=(i > 1), lwd=2, col=cols[i])
legend("bottomright", paste("d =",d.set), bty="n", lwd=2, col=cols)

## Shows the numeric problems:
curve(Vectorize(beta.,"theta")(copFrank, x, d = 29), 35, 42, col="violet")

```

---

## Description

Compute the conditional distribution function  $C(u_d | u_1, \dots, u_{d-1})$  of  $u_d$  given  $u_1, \dots, u_{d-1}$ .

## Usage

```

cCopula(u, copula, indices = 1:dim(copula), inverse = FALSE,
        log = FALSE, drop = FALSE, ...)

```

```
## Deprecated (use cCopula() instead):
rtrafo(u, copula, indices = 1:dim(copula), inverse = FALSE, log = FALSE)
cacopula(u, cop, n.MC = 0, log = FALSE)
```

### Arguments

|             |                                                                                                                                                                                                                                                                                        |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| u           | data <b>matrix</b> in $[0, 1]^{(n, d)}$ of $U(0, 1)^d$ samples if <code>inverse = FALSE</code> and (pseudo-/copula-)observations if <code>inverse = TRUE</code> .                                                                                                                      |
| copula, cop | copula, i.e., an object of class " <b>Copula</b> " with specified parameters; currently, the conditional distribution is only provided for Archimedean and elliptical copulas.                                                                                                         |
| indices     | vector of indices $j$ (in $\{1, \dots, d\}$ ( $d = \text{copula dimension}$ )); unique; sorted in increasing order) for which $C_{j 1, \dots, j-1}(u_j   u_1, \dots, u_{j-1})$ (or, if <code>inverse = TRUE</code> , $C_{j 1, \dots, j-1}^-(u_j   u_1, \dots, u_{j-1})$ ) is computed. |
| inverse     | <b>logical</b> indicating whether the inverse $C_{j 1, \dots, j-1}^-(u_j   u_1, \dots, u_{j-1})$ is returned.                                                                                                                                                                          |
| n.MC        | integer Monte Carlo sample size; for Archimedean copulas only, used if positive.                                                                                                                                                                                                       |
| log         | a <b>logical</b> indicating whether the logarithmic values are returned.                                                                                                                                                                                                               |
| drop        | a <b>logical</b> indicating whether a vector should be returned (instead of a 1-row matrix) when <code>n</code> is 1.                                                                                                                                                                  |
| ...         | additional arguments (currently only used if <code>inverse = TRUE</code> in which case they are passed on to the underlying <code>uniroot()</code> ).                                                                                                                                  |

### Details

By default and if fed with a sample of the corresponding copula, `cCopula()` computes the Rosenblatt transform; see Rosenblatt (1952). The involved high-order derivatives for Archimedean copulas were derived in Hofert et al. (2012).

*Sampling*, that is, random number generation, can be achieved by using `inverse=TRUE`. In this case, the inverse Rosenblatt transformation is used, which, for sampling purposes, is also known as *conditional distribution method*. Note that, for Archimedean copulas not being Clayton, this can be slow as it involves numerical root finding in each (but the first) component.

### Value

An  $(n, k)$ -**matrix** (unless `n == 1` and `drop` is true, where a  $k$ -vector is returned) where  $k$  is the length of indices. This matrix contains the conditional copula function values  $C_{j|1, \dots, j-1}(u_j | u_1, \dots, u_{j-1})$  or, if `inverse = TRUE`, their inverses  $C_{j|1, \dots, j-1}^-(u_j | u_1, \dots, u_{j-1})$  for all  $j$  in indices.

### Note

For some (but not all) families, this function also makes sense on the boundaries (if the corresponding limits can be computed).

### References

Genest, C., Rémillard, B., and Beaudoin, D. (2009). Goodness-of-fit tests for copulas: A review and a power study. *Insurance: Mathematics and Economics* **44**, 199–213.

Rosenblatt, M. (1952). Remarks on a Multivariate Transformation, *The Annals of Mathematical Statistics* **23**, 3, 470–472.

Hofert, M., Mächler, M., and McNeil, A. J. (2012). Likelihood inference for Archimedean copulas in high dimensions under known margins. *Journal of Multivariate Analysis* **110**, 133–150.

### See Also

[htrafo](#); [acopula-families](#).

### Examples

```
## 1) Sampling from a conditional distribution of a Clayton copula given u_1

## Define the copula
tau <- 0.5
theta <- iTau(claytonCopula(), tau = tau)
d <- 2
cc <- claytonCopula(theta, dim = d)
n <- 1000
set.seed(271)

## A small u_1
u1 <- 0.05
U <- cCopula(cbind(u1, runif(n)), copula = cc, inverse = TRUE)
plot(U[,2], ylab = quote(U[2]))

## A large u_1
u1 <- 0.95
U <- cCopula(cbind(u1, runif(n)), copula = cc, inverse = TRUE)
plot(U[,2], ylab = quote(U[2]))

## 2) Sample via conditional distribution method and then apply the
## Rosenblatt transform
## Note: We choose the numerically more involved (and thus slower)
## Gumbel case here

## Define the copula
tau <- 0.5
theta <- iTau(gumbelCopula(), tau = tau)
d <- 5
gc <- gumbelCopula(theta, dim = d)
n <- 200
set.seed(271)
U. <- matrix(runif(n*d), ncol = d) # U(0,1)^d

## Transform to Gumbel sample via conditional distribution method
U <- cCopula(U., copula = gc, inverse = TRUE) # slow for ACs except Clayton
splom2(U) # scatter-plot matrix copula sample

## Rosenblatt transform back to U(0,1)^d (as a check)
```

```

U. <- cCopula(U, copula = gc)
splom2(U.) #  $U(0,1)^d$  again

## 3) cCopula() for elliptical copulas

tau <- 0.5
theta <- iTau(claytonCopula(), tau = tau)
d <- 5
cc <- claytonCopula(theta, dim = d)
set.seed(271)
n <- 1000
U <- rCopula(n, copula = cc)
X <- qnorm(U) # X now follows a meta-Clayton model with  $N(0,1)$  marginals
U <- pobs(X) # build pseudo-observations

fN <- fitCopula(normalCopula(dim = d), data = U) # fit a Gauss copula
U.RN <- cCopula(U, copula = fN@copula)
splom2(U.RN, cex = 0.2) # visible but not so clearly

f.t <- fitCopula(tCopula(dim = d), U)
U.Rt <- cCopula(U, copula = f.t@copula) # transform with a fitted t copula
splom2(U.Rt, cex = 0.2) # still visible but not so clear

## Inverse (and check consistency)
U.N <- cCopula(U.RN, copula = fN @copula, inverse = TRUE)
U.t <- cCopula(U.Rt, copula = f.t@copula, inverse = TRUE)

tol <- 1e-14
stopifnot(
  all.equal(U, U.N),
  all.equal(U, U.t),
  all.equal(log(U.RN),
             cCopula(U, copula = fN @copula, log = TRUE), tolerance = tol),
  all.equal(log(U.Rt),
             cCopula(U, copula = f.t@copula, log = TRUE), tolerance = tol)
)

## 4) cCopula() for a more sophisticated mixture copula (bivariate case only!)

tau <- 0.5
cc <- claytonCopula(iTau(claytonCopula(), tau = tau)) # first mixture component
tc <- tCopula(iTau(tCopula(), tau = tau), df = 3) #  $t_3$  copula
tc90 <- rotCopula(tc, flip = c(TRUE, FALSE)) # t copula rotated by 90 degrees
wts <- c(1/2, 1/2) # mixture weights
mc <- mixCopula(list(cc, tc90), w = wts) # mixture copula with one copula rotated

set.seed(271)
U <- rCopula(n, copula = mc)
U. <- cCopula(U, copula = mc) # Rosenblatt transform back to  $U(0,1)^2$  (as a check)
plot(U., xlab = quote(U*" "[1]), ylab = quote(U*" "[2])) # check for uniformity

```

**Description**

Function and Methods `cloud2()` to draw (**lattice**) **cloud** plots of two-dimensional distributions from package **copula**.

**Usage**

```
## S4 method for signature 'matrix'
cloud2(x,
       xlim = range(x[,1], finite = TRUE),
       ylim = range(x[,2], finite = TRUE),
       zlim = range(x[,3], finite = TRUE),
       xlab = NULL, ylab = NULL, zlab = NULL,
       scales = list(arrows = FALSE, col = "black"),
       par.settings = standard.theme(color = FALSE), ...)
## S4 method for signature 'data.frame'
cloud2(x,
       xlim = range(x[,1], finite = TRUE),
       ylim = range(x[,2], finite = TRUE),
       zlim = range(x[,3], finite = TRUE),
       xlab = NULL, ylab = NULL, zlab = NULL,
       scales = list(arrows = FALSE, col = "black"),
       par.settings = standard.theme(color = FALSE), ...)
## S4 method for signature 'Copula'
cloud2(x, n,
       xlim = 0:1, ylim = 0:1, zlim = 0:1,
       xlab = quote(U[1]), ylab = quote(U[2]), zlab = quote(U[3]), ...)
## S4 method for signature 'mvdc'
cloud2(x, n,
       xlim = NULL, ylim = NULL, zlim = NULL,
       xlab = quote(X[1]), ylab = quote(X[2]), zlab = quote(X[3]), ...)
```

**Arguments**

`x` a "matrix", "data.frame", "Copula" or a "mvdc" object.

`xlim, ylim, zlim` the x-, y- and z-axis limits.

`xlab, ylab, zlab` the x-, y- and z-axis labels.

`scales` a **list** determining how the axes are drawn; see `cloud()`.

`par.settings` see `cloud()`.

`n` when `x` is not matrix-like: The sample size of the random sample drawn from `x`.

`...` additional arguments passed to the underlying `cloud()`.

**Value**

An object of class “trellis” as returned by `cloud()`.

**Methods**

Cloud plots for objects of class “`matrix`”, “`data.frame`”, “`Copula`” or “`mvdc`”.

**See Also**

The **lattice**-based `splom2-methods` for data, and `wireframe2-methods` and `contourplot2-methods` for functions.

**Examples**

```
## For 'matrix' objects
cop <- gumbelCopula(2, dim = 3)
n <- 1000
set.seed(271)
U <- rCopula(n, copula = cop)
cloud2(U, xlab = quote(U[1]), ylab = quote(U[2]), zlab = quote(U[3]))

## For 'Copula' objects
set.seed(271)
cloud2(cop, n = n) # same as above

## For 'mvdc' objects
mvNN <- mvdc(cop, c("norm", "norm", "exp"),
             list(list(mean = 0, sd = 1), list(mean = 1), list(rate = 2)))
cloud2(mvNN, n = n)
```

---

coeffG

*Coefficients of Polynomial used for Gumbel Copula*


---

**Description**

Compute the coefficients  $a_{d,k}(\theta)$  involved in the generator (psi) derivatives and the copula density of Gumbel copulas.

For non-small dimensions  $d$ , these are numerically challenging to compute accurately.

**Usage**

```
coeffG(d, alpha,
       method = c("sort", "horner", "direct", "dsumSibuya",
                  paste("dsSib", eval(formals(dsumSibuya)$method), sep = ".")),
       log = FALSE, verbose = FALSE)
```

**Arguments**

|         |                                                                                                                                                                                                                                                                                                                           |
|---------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| d       | number of coefficients, (the copula dimension), $d \geq 1$ .                                                                                                                                                                                                                                                              |
| alpha   | parameter $1/\theta$ in $(0, 1]$ ; you may use <code>mpfr(alph, precBits = &lt;n_prec&gt;)</code> for higher precision methods ("Rmpfr*") from package <b>Rmpfr</b> .                                                                                                                                                     |
| method  | a <b>character</b> string, one of<br>"sort": compute coefficients via $\exp(\log())$ pulling out the maximum, and sort.<br>"horner": uses polynomial evaluation, our internal <code>polynEval()</code> .<br>"direct": brute force approach.<br>"dsSib.<FOO>": uses <code>dsumSibuya(..., method = "&lt;FOO&gt;")</code> . |
| log     | logical determining if the logarithm ( <code>log</code> ) is to be returned.                                                                                                                                                                                                                                              |
| verbose | logical indicating if some information should be shown, currently for <code>method == "sort"</code> only.                                                                                                                                                                                                                 |

**Value**

a numeric vector of length `d`, of values

$$a_k(\theta, d) = (-1)^{d-k} \sum_{j=k}^d \alpha^j * s(d, j) * S(j, k), k \in \{1, \dots, d\}.$$

**Note**

There are still known numerical problems (with non-"Rmpfr" methods; and those are slow), e.g., for  $d=100$ ,  $\alpha=0.8$  and  $\text{sign}(s(n, k)) = (-1)^{n-k}$ .

As a consequence, the methods and its defaults may change in the future, and so the exact implementation of `coeffG()` is still considered somewhat experimental.

**Examples**

```
a.k <- coeffG(16, 0.55)
plot(a.k, xlab = quote(k), ylab = quote(a[k]),
     main = "coeffG(16, 0.55)", log = "y", type = "o", col = 2)
a.kH <- coeffG(16, 0.55, method = "horner")
stopifnot(all.equal(a.k, a.kH, tol = 1e-11))# 1.10e-13 (64-bit Lnx, nb-mm4)
```

**Description**

Methods for function `contour` to draw contour lines aka a level plot for objects from package **copula**.

**Usage**

```
## S4 method for signature 'Copula'
contour(x, FUN,
        n.grid = 26, delta = 0,
        xlab = quote(u[1]), ylab = quote(u[2]),
        box01 = TRUE, ...)
## S4 method for signature 'mvdc'
contour(x, FUN, xlim, ylim, n.grid = 26,
        xlab = quote(x[1]), ylab = quote(x[2]),
        box01 = FALSE, ...)
```

**Arguments**

|            |                                                                                                                                                                                                                                      |
|------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x          | a "Copula" or a "mvdc" object.                                                                                                                                                                                                       |
| FUN        | the <a href="#">function</a> to be plotted; typically <a href="#">dCopula</a> or <a href="#">pCopula</a> .                                                                                                                           |
| n.grid     | the number of grid points used in each dimension. This can be a vector of length two, giving the number of grid points used in x- and y-direction, respectively; the function FUN will be evaluated on the corresponding (x,y)-grid. |
| delta      | a small number in $[0, \frac{1}{2})$ influencing the evaluation boundaries. The x- and y-vectors will have the range $[\delta, 1-\delta]$ , the default being $[0, 1]$ .                                                             |
| xlab, ylab | the x-axis and y-axis labels.                                                                                                                                                                                                        |
| xlim, ylim | the <a href="#">range</a> of the x and y variables, respectively.                                                                                                                                                                    |
| box01      | a logical specifying if a faint rectangle should be drawn on the boundary of $[0, 1]^2$ (often useful for copulas, but typically <i>not</i> for general multivariate distributions ("mvdc")).                                        |
| ...        | further arguments for (the default method of) <a href="#">contour()</a> , e.g., <a href="#">nlevels</a> , <a href="#">levels</a> , etc.                                                                                              |

**Methods**

Contour lines are drawn for "Copula" or "mvdc" objects, see x in the *Arguments* section.

**See Also**

The [persp-methods](#) for "perspective" aka "3D" plots.

**Examples**

```
contour(frankCopula(-0.8), dCopula)
contour(frankCopula(-0.8), dCopula, delta=1e-6)
contour(frankCopula(-1.2), pCopula)
contour(claytonCopula(2), pCopula)

## the Gumbel copula density is "extreme"
## --> use fine grid (and enough levels):
r <- contour(gumbelCopula(3), dCopula, n=200, nlevels=100)
range(r$z) # [0, 125.912]
```



```
## Now superimpose contours of three resolutions:
contour(r, levels = seq(1, max(r$z), by=2), lwd=1.5)
contour(r, levels = (1:13)/2, add=TRUE, col=adjustcolor(1,3/4), lty=2)
contour(r, levels = (1:13)/4, add=TRUE, col=adjustcolor(2,1/2),
        lty=3, lwd=3/4)

x <- mvdc(gumbelCopula(3), c("norm", "norm"),
          list(list(mean = 0, sd = 1), list(mean = 1)))
contour(x, dMvdc, xlim=c(-2, 2), ylim=c(-1, 3))
contour(x, pMvdc, xlim=c(-2, 2), ylim=c(-1, 3))
```

---

contourplot2-methods *Contour Plot Methods 'contourplot2' in Package 'copula'*

---

## Description

Methods for `contourplot2()`, a version of `contourplot()` from **lattice**, to draw contour plots of two dimensional distributions from package **copula**.

## Usage

```
## NB: The 'matrix' and 'data.frame' methods are identical - documenting the former
## S4 method for signature 'matrix'
contourplot2(x, aspect = 1,
             xlim = range(x[,1], finite = TRUE),
             ylim = range(x[,2], finite = TRUE),
             xlab = NULL, ylab = NULL,
             cuts = 16, labels = !region, pretty = !labels,
             scales = list(alternating = c(1,1), tck = c(1,0)),
             region = TRUE, ...,
             col.regions = gray(seq(0.4, 1, length.out = max(100, 4*cuts))))

## S4 method for signature 'Copula'
contourplot2(x, FUN, n.grid = 26, delta = 0,
             xlim = 0:1, ylim = 0:1,
             xlab = quote(u[1]), ylab = quote(u[2]), ...)

## S4 method for signature 'mvdc'
contourplot2(x, FUN, n.grid = 26, xlim, ylim,
             xlab = quote(x[1]), ylab = quote(x[2]), ...)
```

## Arguments

|                         |                                                                                                                                                                                       |
|-------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>x</code>          | a <i>"matrix"</i> , <i>"data.frame"</i> , <i>"Copula"</i> or a <i>"mvdc"</i> object.                                                                                                  |
| <code>aspect</code>     | the aspect ratio.                                                                                                                                                                     |
| <code>xlim, ylim</code> | the x- and y-axis limits.                                                                                                                                                             |
| <code>xlab, ylab</code> | the x- and y-axis labels. If at least one is NULL, useful <code>xlab</code> and <code>ylab</code> are determined automatically; the behavior depends on the class of <code>x</code> . |

|                |                                                                                                                                                                                                                                                                            |
|----------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| cuts           | the number of levels; see <code>contourplot()</code> . Note that specifying <code>useRaster = TRUE</code> is often considerably more efficient notably for larger values of cuts.                                                                                          |
| labels, pretty | <b>logicals</b> indicating whether the contour lines are labeled and whether pretty labels are enforced; in <b>copula</b> versions before 0.999-18, implicitly <code>pretty = TRUE</code> was used (giving <b>unequal</b> z-cut spacing), see <code>contourplot()</code> . |
| scales         | a <b>list</b> determining how the axes are drawn; see <code>contourplot()</code> .                                                                                                                                                                                         |
| region         | a <b>logical</b> indicating whether regions between contour lines should be filled as in a level plot; see <code>contourplot()</code> .                                                                                                                                    |
| col.regions    | the colors of the regions if <code>region = TRUE</code> ; see <code>contourplot()</code> .                                                                                                                                                                                 |
| FUN            | the <b>function</b> to be plotted; typically <code>dCopula</code> or <code>pCopula</code> .                                                                                                                                                                                |
| n.grid         | the number of grid points used in each dimension. This can be a vector of length two, giving the number of grid points used in x- and y-direction, respectively; the function FUN will be evaluated on the corresponding (x,y)-grid.                                       |
| delta          | a small number in $[0, \frac{1}{2})$ influencing the evaluation boundaries. The x- and y-vectors will have the range $[0+\text{delta}, 1-\text{delta}]$ , the default being $[0, 1]$ .                                                                                     |
| ...            | additional arguments passed to the underlying <code>contourplot()</code> .                                                                                                                                                                                                 |

### Value

An object of class “trellis” as returned by `contourplot()`.

### Methods

Contourplot plots for objects of class `"matrix"`, `"data.frame"`, `"Copula"` or `"mvdc"`.

### See Also

The `contour-methods` for drawing perspective plots via base graphics.

The **lattice**-based `wireframe2-methods` to get perspective plots for functions, and `cloud2-methods` and `splom2-methods` for data.

### Examples

```
## For 'matrix' objects
## The Frechet--Hoeffding bounds W and M
n.grid <- 26
u <- seq(0, 1, length.out = n.grid)
grid <- expand.grid("u[1]" = u, "u[2]" = u)
W <- function(u) pmax(0, rowSums(u)-1) # lower bound W
M <- function(u) apply(u, 1, min) # upper bound M
x.W <- cbind(grid, "W(u[1],u[2])" = W(grid)) # evaluate W on 'grid'
x.M <- cbind(grid, "M(u[1],u[2])" = M(grid)) # evaluate M on 'grid'
contourplot2(x.W) # contour plot of W
contourplot2(x.M) # contour plot of M

## For 'Copula' objects
cop <- frankCopula(-4)
contourplot2(cop, pCopula) # the copula
```

```

contourplot2(cop, pCopula, xlab = "x", ylab = "y") # adjusting the labels
contourplot2(cop, dCopula) # the density

## For 'mvdc' objects
mvNN <- mvdc(gumbelCopula(3), c("norm", "norm"),
             list(list(mean = 0, sd = 1), list(mean = 1)))
x1 <- c(-2, 2)
y1 <- c(-1, 3)
contourplot2(mvNN, FUN = dMvdc, xlim = x1, ylim = y1, contour = FALSE)
contourplot2(mvNN, FUN = dMvdc, xlim = x1, ylim = y1)
contourplot2(mvNN, FUN = dMvdc, xlim = x1, ylim = y1, region = FALSE, labels = FALSE)
contourplot2(mvNN, FUN = dMvdc, xlim = x1, ylim = y1, region = FALSE)
contourplot2(mvNN, FUN = dMvdc, xlim = x1, ylim = y1,
             col.regions = colorRampPalette(c("royalblue3", "maroon3"), space="Lab"))

```

---

copFamilies

*Specific Archimedean Copula Families ("acopula" Objects)*


---

## Description

Specific Archimedean families ("[acopula](#)" objects) implemented in the package **copula**.

These families are “classical” as from p. 116 of Nelsen (2007). More specifically, see Table 1 of Hofert (2011).

## Usage

```

copAMH
copClayton
copFrank
copGumbel
copJoe

```

## Details

All these are objects of the formal class "[acopula](#)".

**copAMH:** Archimedean family of Ali-Mikhail-Haq with parametric generator

$$\psi(t) = (1 - \theta) / (\exp(t) - \theta), \quad t \in [0, \infty],$$

with  $\theta \in [0, 1)$ . The range of admissible Kendall's tau is  $[0, 1/3)$ .

Note that the lower and upper tail-dependence coefficients are both zero, that is, this copula family does not allow for tail dependence.

**copClayton:** Archimedean family of Clayton with parametric generator

$$\psi(t) = (1 + t)^{-1/\theta}, \quad t \in [0, \infty],$$

with  $\theta \in (0, \infty)$ . The range of admissible Kendall's tau, as well as that of the lower tail-dependence coefficient, is  $(0, 1)$ . For dimension  $d = 2$ ,  $\theta \in (-1, \infty)$  is admissible where negative  $\theta$  allow negative Kendall's taus. Note that this copula does not allow for upper tail dependence.

copFrank: Archimedean family of Frank with parametric generator

$$-\log(1 - (1 - e^{-\theta}) \exp(-t))/\theta, t \in [0, \infty]$$

with  $\theta \in (0, \infty)$ . The range of admissible Kendall's tau is (0,1). Note that this copula family does not allow for tail dependence.

copGumbel: Archimedean family of Gumbel with parametric generator

$$\exp(-t^{1/\theta}), t \in [0, \infty]$$

with  $\theta \in [1, \infty)$ . The range of admissible Kendall's tau, as well as that of the upper tail-dependence coefficient, is [0,1). Note that this copula does not allow for lower tail dependence.

copJoe: Archimedean family of Joe with parametric generator

$$1 - (1 - \exp(-t))^{1/\theta}, t \in [0, \infty]$$

with  $\theta \in [1, \infty)$ . The range of admissible Kendall's tau, as well as that of the upper tail-dependence coefficient, is [0,1). Note that this copula does not allow for lower tail dependence.

Note that staying within one of these Archimedean families, all of them can be nested if two (generic) generator parameters  $\theta_0, \theta_1$  satisfy  $\theta_0 \leq \theta_1$ .

### Value

A "acopula" object.

### References

- Nelsen, R. B. (2007). *An Introduction to Copulas* (2nd ed.). Springer.
- Hofert, M. (2010). *Sampling Nested Archimedean Copulas with Applications to CDO Pricing*. Suedwestdeutscher Verlag fuer Hochschulschriften AG & Co. KG.
- Hofert, M. (2011). Efficiently sampling nested Archimedean copulas. *Computational Statistics & Data Analysis* **55**, 57–70.
- Hofert, M. and Mächler, M. (2011). Nested Archimedean Copulas Meet R: The nacopula Package. *Journal of Statistical Software* **39**(9), 1–20. <https://www.jstatsoft.org/v39/i09/>.

### See Also

The class definition, "acopula". [onacopula](#) and [setTheta](#) for such Archimedean copulas with specific parameters.  
[getAcop](#) accesses these families "programmatically".

### Examples

```
## Print a copAMH object and its structure
copAMH
str(copAMH)

## Show admissible parameters for a Clayton copula
copClayton@paraInterval
```

```

## Generate random variates from a Log(p) distribution via V0 of Frank
p <- 1/2
copFrank@V0(100, -log(1-p))

## Plot the upper tail-dependence coefficient as a function in the
## parameter for Gumbel's family
curve(copGumbel@lambdaU(x), xlim = c(1, 10), ylim = c(0,1), col = 4)

## Plot Kendall's tau as a function in the parameter for Joe's family
curve(copJoe@tau(x), xlim = c(1, 10), ylim = c(0,1), col = 4)

## ----- Plot psi() and tau() - and properties of all families ----

## The copula families currently provided:
(famNms <- ls("package:copula", patt="^cop[A-Z]"))

op <- par(mfrow= c(length(famNms), 2),
          mar = .6+ c(2,1.4,1,1), mgp = c(1.1, 0.4, 0))
for(nm in famNms) { Cf <- get(nm)
  thet <- Cf@iTau(0.3)
  curve(Cf@psi(x, theta = thet), 0, 5,
        xlab = quote(x), ylab="", ylim=0:1, col = 2,
        main = substitute(list(NAM ~~~ psi(x, theta == TH), tau == 0.3),
                          list(NAM=Cf@name, TH=thet)))
  I <- Cf@paraInterval
  Iu <- pmin(10, I[2])
  curve(Cf@tau(x), I[1], Iu, col = 3,
        xlab = bquote(theta %in% .(format(I))), ylab = "",
        main = substitute(NAM ~ tau(theta), list(NAM=Cf@name)))
}
par(op)

## Construct a bivariate Clayton copula with parameter theta
theta <- 2
C2 <- onacopula("Clayton", C(theta, 1:2))
C2@copula # is an "acopula" with specific parameter theta

curve(C2@copula@psi(x, C2@copula@theta),
      main = quote("Generator" ~ psi ~ " of Clayton A.copula"),
      xlab = quote(theta1), ylab = quote(psi(theta1)),
      xlim = c(0,5), ylim = c(0,1), col = 4)

## What is the corresponding Kendall's tau?
C2@copula@tau(theta) # 0.5

## What are the corresponding tail-dependence coefficients?
C2@copula@lambdaL(theta)
C2@copula@lambdaU(theta)

## Generate n pairs of random variates from this copula
U <- rnacopula(n = 1000, C2)
## and plot the generated pairs of random variates

```

```
plot(U, asp=1, main = "n = 1000 from Clayton(theta = 2)")
```

---

|        |                                                                               |
|--------|-------------------------------------------------------------------------------|
| Copula | <i>Density, Evaluation, and Random Number Generation for Copula Functions</i> |
|--------|-------------------------------------------------------------------------------|

---

### Description

Density (`dCopula`), distribution function (`pCopula`), and random generation (`rCopula`) for a copula object.

### Usage

```
dCopula(u, copula, log=FALSE, ...)
pCopula(u, copula, ...)
rCopula(n, copula, ...)
```

### Arguments

|                     |                                                                                                                                                                                                                                                                                                             |
|---------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>copula</code> | an R object of class <code>"Copula"</code> , (i.e., <code>"copula"</code> or <code>"nacopula"</code> ).                                                                                                                                                                                                     |
| <code>u</code>      | a vector of the copula dimension $d$ or a matrix with $d$ columns, giving the points where the density or distribution function needs to be evaluated. Note that in all cases, values outside of the cube $[0, 1]^d$ are treated equivalently to those on the cube boundary. So, e.g., the density is zero. |
| <code>log</code>    | logical indicating if the $\log(f(\cdot))$ should be returned instead of $f(\cdot)$ .                                                                                                                                                                                                                       |
| <code>n</code>      | (for <code>rCopula()</code> ): number of observations to be generated.                                                                                                                                                                                                                                      |
| <code>...</code>    | further optional arguments for some methods, e.g., <code>method</code> .                                                                                                                                                                                                                                    |

### Details

The density (`dCopula`) and distribution function (`pCopula`) methods for Archimedean copulas now use the corresponding function slots of the Archimedean copula objects, such as `copClayton`, `copGumbel`, etc.

If an  $u_j$  is outside  $(0, 1)$  we declare the density to be zero, and this is true even when another  $u_k, k \neq j$  is `NA` or `NaN`; see also the “outside” example.

The distribution function of a `t` copula uses `pmvt` from package `mvtnorm`; similarly, the density (`dCopula`) calls `dmvt` from CRAN package `mvtnorm`. The `normalCopula` methods use `dmvnorm` and `pmvnorm` from the same package.

The random number generator for an Archimedean copula uses the conditional approach for the bivariate case and the Marshall-Olkin (1988) approach for dimension greater than 2.

### Value

`dCopula()` gives the density, `pCopula()` gives the distribution function, and `rCopula()` generates random variates.

## References

- Frees, E. W. and Valdez, E. A. (1998). Understanding relationships using copulas. *North American Actuarial Journal* **2**, 1–25.
- Genest, C. and Favre, A.-C. (2007). Everything you always wanted to know about copula modeling but were afraid to ask. *Journal of Hydrologic Engineering* **12**, 347–368.
- Joe, H. (1997). *Multivariate Models and Dependence Concepts*. Chapman and Hall, London.
- Marshall, A. W. and Olkin, I. (1988) Families of multivariate distributions. *Journal of the American Statistical Association* **83**, 834–841.
- Nelsen, R. B. (2006) *An introduction to Copulas*. Springer, New York.

## See Also

the `copula` and `acopula` classes, the `acopula` families, `acopula-families`. Constructor functions such as `ellipCopula`, `archmCopula`, `fgmCopula`.

## Examples

```
norm.cop <- normalCopula(0.5)
norm.cop
## one d-vector =^= 1-row matrix, works too :
dCopula(c(0.5, 0.5), norm.cop)
pCopula(c(0.5, 0.5), norm.cop)

u <- rCopula(100, norm.cop)
plot(u)
dCopula(u, norm.cop)
pCopula(u, norm.cop)
persp (norm.cop, dCopula)
contour(norm.cop, pCopula)

## a 3-dimensional normal copula
u <- rCopula(1000, normalCopula(0.5, dim = 3))
if(require(scatterplot3d))
  scatterplot3d(u)

## a 3-dimensional clayton copula
cl3 <- claytonCopula(2, dim = 3)
v <- rCopula(1000, cl3)
pairs(v)
if(require(scatterplot3d))
  scatterplot3d(v)

## Compare with the "nacopula" version :
fu1 <- dCopula(v, cl3)
fu2 <- copClayton@dacopula(v, theta = 2)
Fu1 <- pCopula(v, cl3)
Fu2 <- pCopula(v, onacopula("Clayton", C(2.0, 1:3)))
## The density and cumulative values are the same:
stopifnot(all.equal(fu1, fu2, tolerance= 1e-14),
          all.equal(Fu1, Fu2, tolerance= 1e-15))
```

```
## NA and "outside" u[]
u <- v[1:12,]
## replace some by values outside (0,1) and some by NA/NaN
u[1, 2:3] <- c(1.5, NaN); u[2, 1] <- 2; u[3, 1:2] <- c(NaN, -1)
u[cbind(4:9, 1:3)] <- c(NaN, NaN)
f <- dCopula(u, c13)
cbind(u, f) # note: f(.) == 0 at [1] and [3] inspite of NaN/NA
stopifnot(f[1:3] == 0, is.na(f[4:9]), 0 < f[10:12])
```

---

copula-class

*Mother Classes "Copula", etc of all Copulas in the Package*


---

## Description

A copula is a multivariate distribution with uniform margins. The virtual class "Copula" is the mother (or "super class") of all copula classes in the package **copula** which encompasses classes of the former packages **nacopula** and **copula**.

The virtual class "parCopula" extends "Copula" and is the super class of all copulas that can be fully *parametrized* and hence fitted to data. For these, at least the `dim()` method must be well defined.

The virtual class "dimCopula" extends "Copula" and has an explicit slot dimension, with corresponding trivial `dim()` method.

The virtual class "copula" extends bot "dimCopula" and "parCopula" and is the mother of all copula classes from former package **copula**. It has set of slots for (the dimension and) parameter vector, see below.

The virtual class "Xcopula" contains a slot copula of class "parCopula".

The virtual class "xcopula" extends "parCopula" and "Xcopula"; an ("actual") class example are the rotated copulas, `rotCopula`.

## Objects from the Class

Objects are typically created by are by `tCopula()`, `evCopula()`, etc.

## Slots

Class "dimCopula" and its subclasses, notably "copula", have a slot  
**dimension:** an "integer" (of length 1), the copula dimension  $d$ .

Class "copula" (and its subclasses) have *additional* slots

**parameters:** **numeric** vector of parameter values, can be NA (i.e., `NA_real_`).

**param.names:** "character" vector of parameter names (and hence of the same length as parameters).

**param.lowbnd:** lower bounds for the parameters, of class "numeric".

**param.upbnd:** upper bounds for the parameters, of class "numeric".

**fullname:** **deprecated**; object of class "character", family names of the copula.



**Warning**

This implementation is still at the experimental stage and is subject to change during the development.

**Note**

The "copula" class is extended by the [evCopula](#), [archmCopula](#), and [ellipCopula](#) classes. Instances of such copulas can be created via functions [evCopula](#), [archmCopula](#) and [ellipCopula](#).

"plackettCopula" and [fgmCopula](#) are special types of copulas which do not belong to either one of the three classes above.

**See Also**

Help for the (sub)classes [archmCopula](#), [ellipCopula](#), [evCopula](#), and [fgmCopula](#).

The Archimedean and nested Archimedean classes (from former package **nacopula**), with a more extensive list of slots (partly instead of methods), [acopula](#), and [nacopula](#).

**Examples**

```
hc <- evCopula("husler", 1.25)
dim(hc)
smoothScatter(u <- rCopula(2^11, hc))
lambda (hc)
tau (hc)
rho(hc)
str(hc)
```

---

corKendall

*(Fast) Computation of Pairwise Kendall's Taus*


---

**Description**

For a data matrix  $x$ , compute the Kendall's tau "correlation" matrix, i.e., all pairwise Kendall's taus between the columns of  $x$ .

By default and when  $x$  has no missing values (NAs), the fast  $O(n \log(n))$  algorithm of [cor.fk\(\)](#) is used.

**Usage**

```
corKendall(x, checkNA = TRUE,
           use = if(checkNA && anyNA(x)) "pairwise" else "everything")
```

**Arguments**

|         |                                                                                                                                                                                                                                                                                       |
|---------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x       | data, a $n \times p$ matrix (or less efficiently a data.frame), or a numeric vector which is treated as $n \times 1$ matrix.                                                                                                                                                          |
| checkNA | logical indicating if x should be checked for NAs and in the case of NA's <i>and</i> when use is not specified ( <i>missing</i> ), <code>cor(*, use = "pairwise")</code> should be used. Note that <code>corKendall(x, checkNA = FALSE)</code> will produce an error when x has NA's. |
| use     | a string to determine the treatment of NAs in x, see <code>cor</code> ; its default determined via checkNA. When this differs from "everything", R's <code>cor</code> is used; otherwise <b>pcaPP</b> 's <code>cor.fk()</code> which cannot deal with NAs.                            |

**Value**

The  $p \times p$  matrix  $K$  of pairwise Kendall's taus, with  $K[i, j] := \text{tau}(x[, i], x[, j])$ .

**See Also**

`cor.fk()` from **pcaPP** (used by default when there are no missing values (NAs) in x).

`etau()` or `fitCopula(*, method = "itau")` make use of `corKendall()`.

**Examples**

```
## If there are no NA's, corKendall() is faster than cor(*, "kendall")
## and gives the same :

system.time(C1 <- cor(swiss, method="kendall"))
system.time(C2 <- corKendall(swiss))
stopifnot(all.equal(C1, C2, tol = 1e-5))

## In the case of missing values (NA), corKendall() reverts to
## cor(*, "kendall", use = "pairwise") {no longer very fast} :

swM <- swiss # shorter names and three missings:
colnames(swM) <- abbreviate(colnames(swiss), min=6)
swM[1,2] <- swM[7,3] <- swM[25,5] <- NA
(C3 <- corKendall(swM)) # now automatically uses the same as
stopifnot(identical(C3, cor(swM, method="kendall", use="pairwise")))
## and is quite close to the non-missing "truth":
stopifnot(all.equal(unname(C3), unname(C2), tol = 0.06)) # rel.diff.= 0.055

try(corKendall(swM, checkNA=FALSE)) # --> Error
## the error is really from pcaPP::cor.fk(swM)
```

dDiag

*Density of the Diagonal of (Nested) Archimedean Copulas***Description**

Evaluate the density of the diagonal of a  $d$ -dimensional (nested) Archimedean copula. Note that the diagonal of a copula is a cumulative distribution function. Currently, only Archimedean copulas are implemented.

**Usage**

```
dDiag(u, cop, log=FALSE)
```

**Arguments**

**u** a numeric vector of evaluation points.

**cop** a (nested) Archimedean copula object of class "[outer\\_nacopula](#)". This also determines the dimension via the `comp` slot

**log** logical indicating if the `log` of the density of the diagonal should be returned instead of just the diagonal density.

**Value**

A **numeric** vector containing the values of the density of the diagonal of the Archimedean copula at `u`.

**References**

Hofert, M., Mächler, M., and McNeil, A. J. (2013). Archimedean Copulas in High Dimensions: Estimators and Numerical Challenges Motivated by Financial Applications. *Journal de la Société Française de Statistique* **154**(1), 25–63.

**See Also**

[acopula](#) class, [dnacopula](#).

**Examples**

```
th. <- c(0.1, 0.2, 0.5, 0.8, 1.4, 2., 5.)
curve(dDiag(x, cop=onacopulaL("Clayton", list(th.[1], 1:3))), 0, 1,
      n=1000, ylab="dDiag(x, *)", main="Diagonal densities of Clayton")
abline(h=0, lty=3)
for(j in 2:length(th.))
  curve(dDiag(x, cop=onacopulaL("Clayton", list(th.[j], 1:3))), add=TRUE,
        col=j, n=1000)
legend("topleft", do.call(expression, lapply(th., function(theta)
      substitute(theta == TH, list(TH=th)))),
      lty = 1, col=seq_along(th.), bty="n")
```

---

describeCop

*Copula (Short) Description as String*


---

**Description**

Describe a [copula](#) object, i.e., its basic properties as a string. This is a utility used when `print()`ing or `plot()`ing copulas, e.g., after a fitting.

**Usage**

```
describeCop(x, kind = c("short", "very short", "long"), prefix = "", ...)
```

**Arguments**

|                     |                                                                                                                                               |
|---------------------|-----------------------------------------------------------------------------------------------------------------------------------------------|
| <code>x</code>      | a <a href="#">copula</a> object, or a generalization such as <a href="#">parCopula</a> .                                                      |
| <code>kind</code>   | a <a href="#">character</a> string specifying the size (or “complexity” of the copula description desired.                                    |
| <code>prefix</code> | a string to be prefixed to the returned string, which can be useful for indentation in describing extended copulas such as Khoudraji copulas. |
| <code>...</code>    | further arguments; unused currently.                                                                                                          |

**Value**

a [character](#) string.

**Methods**

```
signature(x = "archmCopula", kind = "ANY") ..
signature(x = "copula", kind = "character") ..
signature(x = "copula", kind = "missing") ..
signature(x = "ellipCopula", kind = "character") ..
signature(x = "fgmCopula", kind = "ANY") ..
signature(x = "xcopula", kind = "ANY") ..
```

**See Also**

Copula class definition [copula](#);

**Examples**

```
## FIXME
```

**Description**

For a (nested) Archimedean copula (object of class [nacopula](#))  $x$ , `dCopula(u, x)` (or also currently still `dnacopula(x, u)`) evaluates the density of  $x$  at the given vector or matrix  $u$ .

**Usage**

```
## S4 method for signature 'matrix,nacopula'
dCopula(u, copula, log=FALSE, ...)

## *Deprecated*:
dnacopula(x, u, log=FALSE, ...)
```

**Arguments**

`copula, x` an object of class "[outer\\_nacopula](#)".

`u` argument of the copula  $x$ . Note that  $u$  can be a matrix in which case the density is computed for each row of the matrix and the vector of values is returned.

`log` logical indicating if the [log](#) of the density should be returned.

`...` optional arguments passed to the copula's `dacopula` function (slot), such as `n.MC` (non-negative integer) for possible Monte Carlo evaluation (see `dacopula` in [acopula](#)).

**Details**

If it exists, the density of an Archimedean copula  $C$  with generator  $\psi$  at  $\mathbf{u} \in (0, 1)^d$  is given by

$$c(\mathbf{u}) = \psi^{(d)}(\psi^{-1}(u_1) + \dots + \psi^{-1}(u_d)) \prod_{j=1}^d (\psi^{-1}(u_j))' = \frac{\psi^{(d)}(\psi^{-1}(u_1) + \dots + \psi^{-1}(u_d))}{\prod_{j=1}^d \psi'(\psi^{-1}(u_j))}.$$

**Value**

A [numeric](#) vector containing the values of the density of the Archimedean copula at  $u$ .

**Note**

`dCopula(u, copula)` is a *generic* function with methods for *all* our copula classes, see [dCopula](#).

**References**

- Hofert, M., Mächler, M., and McNeil, A. J. (2012). Likelihood inference for Archimedean copulas in high dimensions under known margins. *Journal of Multivariate Analysis* **110**, 133–150.
- Hofert, M., Mächler, M., and McNeil, A. J. (2013). Archimedean Copulas in High Dimensions: Estimators and Numerical Challenges Motivated by Financial Applications. *Journal de la Société Française de Statistique* **154**(1), 25–63.

**See Also**

For more details about the derivatives of an Archimedean generator, see, for example, `absdPsi` in class [acopula](#).

**Examples**

```
## Construct a twenty-dimensional Gumbel copula with parameter chosen
## such that Kendall's tau of the bivariate margins is 0.25.
theta <- copJoe@iTau(.25)
C20 <- onacopula("J", C(theta, 1:20))

## Evaluate the copula density at the point u = (0.5,...,0.5)
u <- rep(0.5, 20)
dCopula(u, C20)

## the same with Monte Carlo based on 10000 simulated "frailties"
dCopula(u, C20, n.MC = 10000)

## Evaluate the exact log-density at several points
u <- matrix(runif(100), ncol=20)
dCopula(u, C20, log = TRUE)

## Back-compatibility check
stopifnot(identical(dCopula(u, C20), suppressWarnings(
  dnacopula(C20, u))),
  identical(dCopula(u, C20, log = TRUE), suppressWarnings(
    dnacopula(C20, u, log = TRUE))))
```

---

ellipCopula

*Construction of Elliptical Copula Class Objects*


---

**Description**

Creating elliptical copula objects with corresponding dimension and parameters, including the dispersion structure  $P$  (pronounced “Rho”).

**Usage**

```
ellipCopula(family, param, dim = 2, dispstr = "ex", df = 4, ...)

normalCopula(param, dim = 2, dispstr = "ex")

  tCopula(param, dim = 2, dispstr = "ex", df = 4,
    df.fixed = FALSE, df.min = 0.01)

dispstrToep(perm = NULL, check = TRUE)

## S4 method for signature 'matrix,normalCopula'
```

```
pCopula(u, copula, algorithm=NULL, keepAttr=FALSE, ...)
## S4 method for signature 'matrix,tCopula'
pCopula(u, copula, algorithm=NULL, keepAttr=FALSE, ...)
```

### Arguments

|           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
|-----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| family    | a <b>character</b> string specifying the family of an elliptical copula. Must be "normal" (the default) or "t".                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| param     | a <b>numeric</b> vector specifying the parameter values; P2p() accesses this vector, whereas p2P() and getSigma() provide the corresponding "P" matrix, see below.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| dim       | the dimension of the copula.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| dispstr   | a string specifying the "dispersion structure", i.e., type of the symmetric positive definite matrix characterizing the elliptical copula. Currently available structures are "ex" for <b>exchangeable</b> , "ar1" for $AR(1)$ , "toep" for Toeplitz ( <b>toeplitz</b> ), and "un" for <b>unstructured</b> .<br>The dispersion structure for Toeplitz can (and often should) now be specified by dispstrToep(), see there.                                                                                                                                                                                                                                                    |
| df        | integer value specifying the number of degrees of freedom of the multivariate t distribution used to construct the t copulas.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| df.fixed  | logical specifying if the degrees of freedom df will be considered as a parameter (to be estimated) or not. The default, FALSE, means that df is to be estimated if the object is passed as argument to fitCopula.                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| df.min    | non-negative number; the strict lower bound for df, mainly during fitting when df.fixed=FALSE, with fitCopula.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| copula    | an R object of class "Copula", in our case inheriting from "ellipCopula".                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |
| u         | a vector of the copula dimension $d$ or a matrix with $d$ columns, giving the points where the distribution function needs to be evaluated. Note that values outside of the cube $[0, 1]^d$ are treated equivalently to those on the cube boundary.                                                                                                                                                                                                                                                                                                                                                                                                                           |
| algorithm | NULL or an "algorithm" object for package <b>mvtnorm</b> 's pmvt() or pmvnorm() functions, see <b>algorithms</b> . Note that for larger dimensions, the monte-carlo based GenzBretz(..) must be used, consequently with slightly random results. By default, algorithm = NULL, algorithm is chosen separately for each row $x <- u[i,]$ , for<br><br>normalCopula: via hidden function pmvnormAlgo(dim, x, ...) which currently is defined as<br><pre>pmvnormAlgo &lt;- function(dim, x, ...) {   if(dim &lt;= 3 &amp;&amp; !anyNA(x) &amp;&amp; (!any(xI &lt;- x == Inf)    all(xI)))     TVPACK(...)   else if(dim &lt;= 5)     Miwa(...)   else     GenzBretz(...) }</pre> |

**tCopula:** via (hidden) `pmvtAlgo(dim, x, ...)` which is the same as `pmvnormAlgo()` above, but as `Miwa()` is not applicable, without the `else if(dim <= 5)` `Miwa(...)` clause.

**keepAttr** `logical` passed to `pmvnorm` or `pmvt`, respectively.

**...** for the `pCopula()` methods, optional arguments to the corresponding algorithm.

**perm** an `integer` vector of length  $d = \text{dim}$ , which must be a permutation of  $1:d$  specifying the (column) ordering of the variables which has Toeplitz dispersion.

**check** a `logical` specifying if the validity of `perm` should be checked (not strictly, currently).

### Value

For

`ellipCopula()`, `normalCopula()`, or `tCopula()`: an elliptical copula object of class "`normalCopula`" or "`tCopula`".

**dispstrToep():** the `character` string "toep", optionally with attribute (see `attributes`, `attr`) "perm" with a permutation  $p$  of  $1:d$ , such that (the column permuted)  $U_p$ , or in the data case the column-permuted matrix  $U[,p]$  has as dispersion matrix `toeplitz(c(1, par))`, with `par` the respective parameter vector of bivariate "correlations"  $\rho_{i,j}$ .

Note that the result of `dispstrToep()` is currently stored in the `dispstr` slot of the copula object.

**pCopula(u, \*):** the numerical vector of copula probabilities of length `nrow(u)`.

### Note

`ellipCopula()` is a wrapper for `normalCopula()` and `tCopula()`.

The `pCopula()` methods for the normal- and t-copulas accept optional arguments to be passed to the underlying (numerical integration) algorithms from package `mvtnorm`'s `pmvnorm` and `pmvt`, respectively, notably `algorithm`, see `GenzBretz`, or `abseps` which defaults to `0.001`.

### See Also

`p2P()`, and `getSigma()` for construction and extraction of the dispersion matrix  $P$  or *Sigma* matrix of (generalized) correlations.

`archmCopula`, `fitCopula`.

### Examples

```
normalCopula(c(0.5, 0.6, 0.7), dim = 3, dispstr = "un")
t.cop <- tCopula(c(0.5, 0.3), dim = 3, dispstr = "toep",
               df = 2, df.fixed = TRUE)
getSigma(t.cop) # P matrix (with diagonal = 1)
stopifnot(all.equal(toeplitz(c(1, .5, .3)), getSigma(t.cop)))

## dispersion "AR1" :
nC.7 <- normalCopula(0.8, dim = 7, dispstr = "ar1")
getSigma(nC.7)
```



```

stopifnot(all.equal(toeplitz(.8^(0:6)), getSigma(nC.7)))

## from the wrapper
norm.cop <- ellipCopula("normal", param = c(0.5, 0.6, 0.7),
  dim = 3, dispstr = "un")
if(require("scatterplot3d") && dev.interactive(orNone=TRUE)) {
  ## 3d scatter plot of 1000 random observations
  scatterplot3d(rCopula(1000, norm.cop))
  scatterplot3d(rCopula(1000, t.cop))
}
set.seed(12)
uN <- rCopula(512, norm.cop)
set.seed(2); pN1 <- pCopula(uN, norm.cop)
set.seed(3); pN2 <- pCopula(uN, norm.cop)
stopifnot(identical(pN1, pN2)) # no longer random for dim = 3
(Xtras <- copula::doExtras())
if(Xtras) { ## a bit more accurately:
  set.seed(4); pN1. <- pCopula(uN, norm.cop, abseps = 1e-9)
  set.seed(5); pN2. <- pCopula(uN, norm.cop, abseps = 1e-9)
  stopifnot(all.equal(pN1., pN2., 1e-5))# see 3.397e-6
  ## but increasing the required precision (e.g., abseps=1e-15) does *NOT* help
}

## For smaller copula dimension 'd', alternatives are available and
## non-random, see ?GenzBretz from package 'mvtnorm' :
has_mvtn <- "package:mvtnorm" %in% search() ## << (workaround ESS Rd render bug)
if(!has_mvtn)
  require("mvtnorm")# -> GenzBretz(), Miva(), and TVPACK() are available
## Note that Miwa() would become very slow for dimensions 5, 6, ..
set.seed(4); pN1.M <- pCopula(uN, norm.cop, algorithm = Miwa(steps = 512))
set.seed(5); pN2.M <- pCopula(uN, norm.cop, algorithm = Miwa(steps = 512))
stopifnot(all.equal(pN1.M, pN2.M, tol= 1e-15))# *no* randomness
set.seed(4); pN1.T <- pCopula(uN, norm.cop, algorithm = TVPACK(abseps = 1e-10))
set.seed(5); pN2.T <- pCopula(uN, norm.cop, algorithm = TVPACK(abseps = 1e-14))
stopifnot(all.equal(pN1.T, pN2.T, tol= 1e-15))# *no* randomness (but no effect of 'abseps')
if(!has_mvtn)
  detach("package:mvtnorm")# (revert)

## Versions with unspecified parameters:
tCopula()
allEQ <- function(u,v) all.equal(u, v, tolerance=0)
stopifnot(allEQ(ellipCopula("norm"), normalCopula()),
  allEQ(ellipCopula("t"), tCopula()))
tCopula(dim=3)
tCopula(dim=4, df.fixed=TRUE)
tCopula(dim=5, disp = "toep", df.fixed=TRUE)
normalCopula(dim=4, disp = "un")

## Toeplitz after *permutation* dispersions (new in copula 1.1-0) -----
tpar <- c(7,5,3)/8 # *gives* pos.def.:
(ev <- eigen(toeplitz(c(1, tpar))), symmetric=TRUE, only.values=TRUE)$values)
stopifnot(ev > 0)

```

```

N4.  <- ellipCopula("normal", dim=4, param=tpar, dispstr = "toep") #"regular"
## reversed order is "the same" for toeplitz structure:
N4.pr <- ellipCopula("normal", dim=4, param=tpar, dispstr = dispstrToep(4:1))
N4.p1 <- ellipCopula("normal", dim=4, param=tpar, dispstr = dispstrToep(c(4,1:3)))
N4.p2 <- ellipCopula("normal", dim=4, param=tpar, dispstr = dispstrToep(c(4:3,1:2)))
N4.p3 <- ellipCopula("normal", dim=4, param=tpar, dispstr = dispstrToep(c(2,4,1,3)))

(pm <- attr(N4.p3@dispstr, "perm")) # (2 4 1 3)
ip <- c(3,1,4,2) # the *inverse* permutation of (2 4 1 3) = Matrix::invPerm(pm)
(Sp3 <- getSigma(N4.p3)) # <-- "permuted toeplitz"
Sp3[ip, ip] # re-ordered rows & columns => *is* toeplitz :
stopifnot(exprs = {
  ## permutations pm and ip are inverses:
  pm[ip] == 1:4
  ip[pm] == 1:4
  is.matrix(T4 <- toeplitz(c(1, tpar)))
  identical(getSigma(N4.), T4)
  identical(getSigma(N4.pr), T4) # 4:1 and 1:4 is "the same" for Rho
  identical(Sp3[ip, ip] , T4)
  identical(Sp3, T4[pm,pm])
})
## Data generation -- NB: The U matrices are equal only "in distribution":
set.seed(7); U.p3 <- rCopula(1000, N4.p3)
set.seed(7); U. <- rCopula(1000, N4.)
stopifnot(exprs = {
  all.equal(loglikCopula(tpar, u=U.p3, copula= N4.p3),
            loglikCopula(tpar, u=U.p3[,ip], copula= N4.) -> LL3)
  all.equal(loglikCopula(tpar, u=U., copula= N4.),
            loglikCopula(tpar, u=U.[,pm], copula= N4.p3) -> LL.)
})
c(LL. , LL3)# similar but different
if(Xtras) {
  fm. <- fitCopula(N4. , U. )
  fm.3 <- fitCopula(N4.p3, U.p3)
  summary(fm.3)
  stopifnot(all.equal(coef(fm.), coef(fm.3), tol = 0.01))# similar but different
}

```

---

ellipCopula-class

*Class "ellipCopula" of Elliptical Copulas*


---

## Description

Copulas generated from elliptical multivariate distributions, notably Normal- and t-copulas (of specific class "normalCopula" or "tCopula", respectively).

## Objects from the Class

Objects are typically created by `ellipCopula()`, `normalCopula()`, or `tCopula()`.

**Slots**

dispstr: "character" string indicating how the dispersion matrix is parameterized; one of "ex", "ar1", "toep", or "un", see the dispstr argument of `ellipCopula()`.

dimension: Object of class "numeric", dimension of the copula.

parameters: a `numeric`, (vector of) the parameter value(s).

param.names: `character` vector with names for the parameters slot, of the same length.

param.lowbnd: `numeric` vector of lower bounds for the parameters slot, of the same length.

param.upbnd: upper bounds for parameters, analogous to `param.lowbnd`.

fullname: **deprecated**; object of class "character", family names of the copula.

**Extends**

Class "ellipCopula" extends class `copula` directly. Classes "normalCopula" and "tCopula" extend "ellipCopula" directly.

**Methods**

Many methods are available, notably `dCopula`, `pCopula`, and `rCopula`. Use, e.g., `methods(class = "tCopula")` to find others.

**See Also**

`ellipCopula` which also documents `tCopula()` and `normalCopula()`; `copula-class`.

---

 emde

---

*Minimum Distance Estimators for (Nested) Archimedean Copulas*


---

**Description**

Compute minimum distance estimators for (nested) Archimedean copulas.

**Usage**

```
emde(u, cop,
     method=c("mde.chisq.CvM", "mde.chisq.KS",
              "mde.gamma.CvM", "mde.gamma.KS"),
     interval=initOpt(cop@copula@name),
     include.K = FALSE, repara = TRUE, ...)
```

**Arguments**

|           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|-----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| u         | $n \times d$ -matrix of (pseudo-)observations (each value in $[0, 1]$ ) from the copula, where $n$ denotes the sample size and $d$ the dimension.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| cop       | <a href="#">outer_nacopula</a> to be estimated (currently only Archimedean copulas are provided).                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| method    | a <a href="#">character</a> string specifying the distance method, which has to be one (or a unique abbreviation) of<br><p>"mde.chisq.CvM" map to an Erlang distribution and using a chi-square distribution and Cramér-von Mises distance;<br/> "mde.chisq.KS" map to an Erlang distribution and using a chi-square distribution and Kolmogorov-Smirnov distance;<br/> "mde.gamma.CvM" map to an Erlang distribution and using a Erlang distribution and Cramér-von Mises distance;<br/> "mde.gamma.KS" map to an Erlang distribution and using a Kolmogorov-Smirnov distance.</p> The four methods are described in Hofert et al. (2013); see also the ‘Details’ section. |
| interval  | bivariate vector denoting the interval where optimization takes place. The default is computed as described in Hofert et al. (2013).                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| include.K | logical indicating whether the last component, the (possibly numerically challenging) Kendall distribution function $K$ , is used ( <code>include.K=TRUE</code> ) or not. Note that the default is <code>FALSE</code> here, where it is <code>TRUE</code> in the underlying <a href="#">htrafo()</a> function.                                                                                                                                                                                                                                                                                                                                                              |
| repara    | logical indicating whether the distance function to be optimized is reparametrized (the default); see the code for more details.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| ...       | additional arguments passed to <a href="#">optimize()</a> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |

**Details**

First, [htrafo](#) is applied to map the  $n \times d$ -matrix of given realizations to a  $n \times d$ -matrix or  $n \times (d-1)$ -matrix, depending on whether the last component is included (`include.K=TRUE`) or not. Second, using either the sum of squares of the standard normal quantile function (`method="mde.chisq.CvM"` and `method="mde.chisq.KS"`) or the sum of negative logarithms (`method="mde.gamma.CvM"` and `method="mde.gamma.KS"`), a map to a chi-square or an Erlang distribution is applied, respectively. Finally, a Cramér-von Mises (`method="mde.chisq.CvM"` and `method="mde.gamma.CvM"`) or Kolmogorov-Smirnov (`method="mde.chisq.KS"` and `method="mde.gamma.KS"`) distance is applied. This is repeated in an optimization until the copula parameter is found such that this distance is minimized.

Note that the same transformations as described above are applied for goodness-of-fit testing; see the ‘See Also’ section).

**Value**

[list](#) as returned by [optimize](#), including the minimum distance estimator.

## References

Hofert, M., Mächler, M., and McNeil, A. J. (2013). Archimedean Copulas in High Dimensions: Estimators and Numerical Challenges Motivated by Financial Applications. *Journal de la Société Française de Statistique* **154**(1), 25–63.

Hering, C. and Hofert, M. (2014), Goodness-of-fit tests for Archimedean copulas in high dimensions, *Innovations in Quantitative Risk Management*.

## See Also

[enacopula](#) (wrapper for different estimators), [gofCopula](#) (wrapper for different goodness-of-fit tests), [htrafo](#) (transformation to a multivariate uniform distribution), and [K](#) (Kendall distribution function).

## Examples

```
tau <- 0.25
(theta <- copGumbel@iTau(tau)) # 4/3
d <- 20
(cop <- onacopulaL("Gumbel", list(theta,1:d)))

set.seed(1)
n <- 200
U <- rnacopula(n, cop)

(meths <- eval(formals(emle)$method)) # "mde.chisq.CvM", ...
fun <- function(meth, u, cop, theta){
  run.time <- system.time(val <- emde(u, cop=cop, method=meth)$minimum)
  list(value=val, error=val-theta, utime.ms=1000*run.time[[1]])
}
(res <- sapply(meths, fun, u=U, cop=cop, theta=theta))
```

---

emle

*Maximum Likelihood Estimators for (Nested) Archimedean Copulas*


---

## Description

Compute (simulated) maximum likelihood estimators for (nested) Archimedean copulas.

## Usage

```
emle(u, cop, n.MC=0, optimizer="optimize", method,
     interval=initOpt(cop@copula@name),
     start=list(theta=initOpt(cop@copula@name, interval=FALSE, u=u)),
     ...)
.emle(u, cop, n.MC=0,
      interval=initOpt(cop@copula@name), ...)
```

## Arguments

|           |                                                                                                                                                                                                                                                                                                                                                                            |
|-----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| u         | $n \times d$ -matrix of (pseudo-)observations (each value in $[0, 1]$ ) from the copula, with $n$ the sample size and $d$ the dimension.                                                                                                                                                                                                                                   |
| cop       | <a href="#">outer_nacopula</a> to be estimated (currently only non-nested, that is, Archimedean copulas are admitted).                                                                                                                                                                                                                                                     |
| n.MC      | <a href="#">integer</a> , if positive, <i>simulated</i> maximum likelihood estimation (SMLE) is used with sample size equal to n.MC; otherwise (n.MC=0), MLE. In SMLE, the $d$ th generator derivative and thus the copula density is evaluated via (Monte Carlo) simulation, whereas MLE uses the explicit formulas for the generator derivatives; see the details below. |
| optimizer | a string or NULL, indicating the optimizer to be used, where NULL means to use <a href="#">optim</a> via the standard R function <a href="#">mle()</a> from (base R) package <b>stats4</b> , whereas the default, "optimize" uses <a href="#">optimize</a> via the R function <a href="#">mle2()</a> from package <b>bbmle</b> .                                           |
| method    | only when optimizer is NULL or "optim", the method to be used for <a href="#">optim</a> .                                                                                                                                                                                                                                                                                  |
| interval  | bivariate vector denoting the interval where optimization takes place. The default is computed as described in Hofert et al. (2012).                                                                                                                                                                                                                                       |
| start     | <a href="#">list</a> of initial values, passed through.                                                                                                                                                                                                                                                                                                                    |
| ...       | additional parameters passed to <a href="#">optimize</a> .                                                                                                                                                                                                                                                                                                                 |

## Details

Exact formulas for the generator derivatives were derived in Hofert et al. (2012). Based on these formulas one can compute the (log-)densities of the Archimedean copulas. Note that for some densities, the formulas are numerically highly non-trivial to compute and considerable efforts were put in to make the computations numerically feasible even in large dimensions (see the source code of the Gumbel copula, for example). Both MLE and SMLE showed good performance in the simulation study conducted by Hofert et al. (2013) including the challenging 100-dimensional case. Alternative estimators (see also [enacopula](#)) often used because of their numerical feasibility, might break down in much smaller dimensions.

Note: SMLE for Clayton currently faces serious numerical issues and is due to further research. This is only interesting from a theoretical point of view, since the exact derivatives are known and numerically non-critical to evaluate.

## Value

**emle** an R object of class "mle2" (and thus useful for obtaining confidence intervals) with the (simulated) maximum likelihood estimator.

**.emle** [list](#) as returned by [optimize\(\)](#) including the maximum likelihood estimator (does not confidence intervals but is typically faster).

## References

Hofert, M., Mächler, M., and McNeil, A. J. (2012). Likelihood inference for Archimedean copulas in high dimensions under known margins. *Journal of Multivariate Analysis* **110**, 133–150.

Hofert, M., Mächler, M., and McNeil, A. J. (2013). Archimedean Copulas in High Dimensions: Estimators and Numerical Challenges Motivated by Financial Applications. *Journal de la Société Française de Statistique* **154**(1), 25–63.

### See Also

`mle2` from package `bbmle` and `mle` from `stats4` on which `mle2` is modeled. `enacopula` (wrapper for different estimators). `demo`(opC-demo) and `demo`(GIG-demo) for examples of two-parameter families.

### Examples

```
tau <- 0.25
(theta <- copGumbel@iTau(tau)) # 4/3
d <- 20
(cop <- onacopulaL("Gumbel", list(theta,1:d)))

set.seed(1)
n <- 200
U <- rnacopula(n,cop)

## Estimation
system.time(efm <- emle(U, cop))
summary(efm) # using bblme's 'mle2' method

## Profile likelihood plot [using S4 methods from bbmle/stats4] :
pfm <- profile(efm)
ci <- confint(pfm, level=0.95)
ci
stopifnot(ci[1] <= theta, theta <= ci[2])
plot(pfm) # |z| against theta, |z| = sqrt(deviance)
plot(pfm, absVal=FALSE, # z against theta
      show.points=TRUE) # showing how it's interpolated
## and show the true theta:
abline(v=theta, col="lightgray", lwd=2, lty=2)
axis(1, pos = 0, at=theta, label=quote(theta[0]))

## Plot of the log-likelihood, MLE and conf.int.:
logL <- function(x) -efm@minuslogl(x)
# == -sum(copGumbel@dacopula(U, theta=x, log=TRUE))
logL. <- Vectorize(logL)
I <- c(cop@copula@iTau(0.1), cop@copula@iTau(0.4))
curve(logL., from=I[1], to=I[2], xlab=quote(theta),
      ylab="log-likelihood",
      main="log-likelihood for Gumbel")
abline(v = c(theta, efm@coef), col="magenta", lwd=2, lty=2)
axis(1, at=c(theta, efm@coef), padj = c(-0.5, -0.8), hadj = -0.2,
      col.axis="magenta", label= expression(theta[0], hat(theta)[n]))
abline(v=ci, col="gray30", lwd=2, lty=3)
text(ci[2], extendrange(par("usr")[3:4], f= -.04)[1],
      "95% conf. int.", col="gray30", adj = -0.1)
```

empCopula

*The Empirical Copula***Description**

Computes the empirical copula (according to a provided method) and auxiliary tools.

**Usage**

```
empCopula(X, smoothing = c("none", "beta", "checkerboard",
                          "schaake.shuffle"), offset = 0,
          ties.method = c("max", "average", "first", "last", "random", "min"))
C.n(u, X, smoothing = c("none", "beta", "checkerboard"), offset = 0,
    ties.method = c("max", "average", "first", "last", "random", "min"))
dCn(u, U, j.ind = 1:d, b = 1/sqrt(nrow(U)), ...)
F.n(x, X, offset = 0, smoothing = c("none", "beta", "checkerboard"))
Cn(x, w) ## <-- deprecated! use C.n(w, x) instead!
toEmpMargins(U, x, ...)
```

**Arguments**

|                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|--------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>X</code>           | an $(n, d)$ - <b>matrix</b> of pseudo-observations with $d$ columns (as <code>x</code> or <code>u</code> ). Recall that a multivariate random sample can be transformed to pseudo-observations via <code>pobs()</code> . For <code>F.n()</code> and if <code>smoothing != "none"</code> , <code>X</code> can also be a general, multivariate sample, in which case the empirical distribution function is computed.                                                                                                                                                                                                                                                                                     |
| <code>u, w</code>        | an $(m, d)$ - <b>matrix</b> with elements in $[0, 1]$ whose rows contain the evaluation points of the empirical copula.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| <code>U</code>           | an $(n, d)$ - <b>matrix</b> of pseudo- (or copula-)observations (elements in $[0, 1]$ , same number $d$ of columns as <code>u</code> (for <code>dCn()</code> ) or <code>x</code> (for <code>toEmpMargins()</code> ).                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| <code>x</code>           | an $(m, d)$ - <b>matrix</b> whose rows contain the evaluation points of the empirical distribution< function (if <code>smoothing = "none"</code> ) or copula (if <code>smoothing != "none"</code> ).                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| <code>smoothing</code>   | <b>character</b> string specifying the type of smoothing of the empirical distribution function (for <code>F.n()</code> ) or the empirical copula (for <code>C.n()</code> ). Available are:<br>"none" the original empirical distribution function or empirical copula.<br>"beta" the empirical beta smoothed distribution function or empirical beta copula.<br>"checkerboard" empirical checkerboard construction.<br>"schaake.shuffle" in each dimension, $n$ (so <code>nrow(X)</code> -many) sorted standard uniforms are used to construct a smooth sample, from which one draws with replacement as many observations as required; only available for the empirical copula and only for sampling. |
| <code>ties.method</code> | <b>character</b> string specifying how ranks should be computed if there are ties in any of the coordinate samples of <code>x</code> ; passed to <code>pobs</code> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| <code>j.ind</code>       | <b>integer</b> vector of indices $j$ between 1 and $d$ indicating the dimensions with respect to which first-order partial derivatives are approximated.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |



|        |                                                                                                                    |
|--------|--------------------------------------------------------------------------------------------------------------------|
| b      | numeric giving the bandwidth for approximating first-order partial derivatives.                                    |
| offset | used in scaling the result which is of the form $\text{sum}(\dots)/(\text{n+offset})$ ; defaults to zero.          |
| ...    | additional arguments passed to <code>dCn()</code> or <code>sort()</code> underlying to <code>EmpMargins()</code> . |

## Details

Given pseudo-observations from a distribution with continuous margins and copula  $C$ , the *empirical copula* is the (default) empirical distribution function of these pseudo-observations. It is thus a natural nonparametric estimator of  $C$ . The function `C.n()` computes the empirical copula or two alternative smoothed versions of it: the *empirical beta copula* or the *empirical checkerboard copula*; see Eqs. (2.1) and (4.1) in Segers, Sibuya and Tsukahara (2017), and the references therein. `empCopula()` is the constructor of an object of class `empCopula`.

The function `dCn()` approximates first-order partial derivatives of the unknown copula using the empirical copula.

The function `F.n()` computes the empirical distribution function of a multivariate sample. Note that `C.n(u, X, smoothing="none", *)` simply calls `F.n(u, pobs(X), *)` after checking  $u$ .

There are several asymptotically equivalent definitions of the empirical copula. `C.n(, smoothing = "none")` is simply defined as the empirical distribution function computed from the pseudo-observations, that is,

$$C_n(\mathbf{u}) = \frac{1}{n} \sum_{i=1}^n \mathbf{1}_{\{\hat{U}_i \leq \mathbf{u}\}},$$

where  $\hat{U}_i$ ,  $i \in \{1, \dots, n\}$ , denote the pseudo-observations and  $n$  the sample size. Internally, `C.n(, smoothing = "none")` is just a wrapper for `F.n()` and is expected to be fed with the pseudo-observations.

The approximation for the  $j$ th partial derivative of the unknown copula  $C$  is implemented as, for example, in Rémillard and Scaillet (2009), and given by

$$\hat{C}_{jn}(\mathbf{u}) = \frac{C_n(u_1, \dots, u_{j-1}, \min(u_j + b, 1), u_{j+1}, \dots, u_d) - C_n(u_1, \dots, u_{j-1}, \max(u_j - b, 0), u_{j+1}, \dots, u_d)}{2b},$$

where  $b$  denotes the bandwidth and  $C_n$  the empirical copula.

## Value

`empCopula()` is the constructor for objects of class `empCopula`.

`C.n()` returns the empirical copula of the pseudo-observations  $X$  evaluated at  $u$  (or a smoothed version of it).

`dCn()` returns a vector (`length(j.ind)` is 1) or a matrix (with number of columns equal to `length(j.ind)`), containing the approximated first-order partial derivatives of the unknown copula at  $u$  with respect to the arguments in `j.ind`.

`F.n()` returns the empirical distribution function of  $X$  evaluated at  $x$  if `smoothing = "none"`, the empirical beta copula evaluated at  $x$  if `smoothing = "beta"` and the empirical checkerboard copula evaluated at  $x$  if `smoothing = "checkerboard"`.

`toEmpMargins()` transforms the copula sample  $U$  to the empirical margins based on the sample  $x$ .

**Note**

The first version of our empirical copula implementation, `Cn()`, had its two arguments *reversed* compared to `C.n()`, and is deprecated now. You **must** swap its arguments to transform to new code.

The use of the two smoothed versions assumes implicitly no ties in the component samples of the data.

**References**

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- Kiriliouk, A., Segers, J. and Tsukahara, H. (2020). Resampling Procedures with Empirical Beta Copulas. <https://arxiv.org/abs/1905.12466>.

**See Also**

[pobs\(\)](#) for computing pseudo-observations.

**Examples**

```
## Generate data X (from a meta-Gumbel model with N(0,1) margins)
n <- 100
d <- 3
family <- "Gumbel"
theta <- 2
cop <- onacopulaL(family, list(theta=theta, 1:d))
set.seed(1)
X <- qnorm(rCopula(n, cop)) # meta-Gumbel data with N(0,1) margins

## Evaluate empirical copula
u <- matrix(runif(n*d), n, d) # random points were to evaluate the empirical copula
ec <- C.n(u, X = X)

## Compare the empirical copula with the true copula
pc <- pCopula(u, copula = cop)
mean(abs(pc - ec)) # ~ 0.012 -- increase n to decrease this error

## The same for the two smoothed versions
```

```

beta <- C.n(u, X, smoothing = "beta")
mean(abs(pc - beta))
check <- C.n(u, X, smoothing = "checkerboard")
mean(abs(pc - check))

## Compare the empirical copula with F.n(pobs())
U <- pobs(X) # pseudo-observations
stopifnot(identical(ec, F.n(u, X = pobs(U)))) # even identical

## Compare the empirical copula based on U at U with the Kendall distribution
## Note: Theoretically, C(U) ~ K, so K(C_n(U, U = U)) should approximately be U(0,1)
plot(ecdf(pK(C.n(U, X), cop = cop@copula, d = d)), asp = 1, xaxs="i", yaxs="i")
segments(0,0, 1,1, col=adjustcolor("blue",1/3), lwd=5, lty = 2)
abline(v=0:1, col="gray70", lty = 2)

## Compare the empirical copula and the true copula on the diagonal
C.n.diag <- function(u) C.n(do.call(cbind, rep(list(u), d)), X = X) # diagonal of C_n
C.diag <- function(u) pCopula(do.call(cbind, rep(list(u), d)), cop) # diagonal of C
curve(C.n.diag, from = 0, to = 1, # empirical copula diagonal
      main = paste("True vs empirical diagonal of a", family, "copula"),
      xlab = "u", ylab = quote("True C(u,..,u) and empirical"~C[n](u,..,u)))
curve(C.diag, lty = 2, add = TRUE) # add true copula diagonal
legend("bottomright", lty = 2:1, bty = "n", inset = 0.02,
      legend = expression(C, C[n]))

## Approximate partial derivatives w.r.t. the 2nd and 3rd component
j.ind <- 2:3 # indices w.r.t. which the partial derivatives are computed
## Partial derivatives based on the empirical copula and the true copula
der23 <- dCn(u, U = pobs(U), j.ind = j.ind)
der23. <- copula::dCdu(archmCopula(family, param=theta, dim=d), u=u)[,j.ind]
## Approximation error
summary(as.vector(abs(der23-der23.)))

## For an example of using F.n(), see help(mvdc)% ./Mvdc.Rd

## Generate a bivariate empirical copula object (various smoothing methods)
n <- 10 # sample size
d <- 2 # dimension
set.seed(271)
X <- rCopula(n, copula = claytonCopula(3, dim = d))
ecop.orig <- empCopula(X) # smoothing = "none"
ecop.beta <- empCopula(X, smoothing = "beta")
ecop.check <- empCopula(X, smoothing = "checkerboard")

## Sample from these (smoothed) empirical copulas
m <- 50
U.orig <- rCopula(m, copula = ecop.orig)
U.beta <- rCopula(m, copula = ecop.beta)
U.check <- rCopula(m, copula = ecop.check)

## Plot
wireframe2(ecop.orig, FUN = pCopula, draw.4.pCoplins = FALSE)
wireframe2(ecop.beta, FUN = pCopula)

```

```

wireframe2(ecop.check, FUN = pCopula)
## Density (only exists when smoothing = "beta")
wireframe2(ecop.beta, FUN = dCopula)

## Transform a copula sample to empirical margins
set.seed(271)
X <- qexp(rCopula(1000, copula = claytonCopula(2))) # multivariate distribution
U <- rCopula(917, copula = gumbelCopula(2)) # new copula sample
X. <- toEmpMargins(U, x = X) # tranform U to the empirical margins of X
plot(X.) # Gumbel sample with empirical margins of X

```

---

empCopula-class      *Class "empCopula" of Empirical Copulas*

---

### Description

Empirical Copula class.

### Objects from the Class

Created by calls of the form `new("empCopula", ...)` or rather typically by `empCopula()` based on a matrix `X` of pseudo-observations. Smoothing options are available, see there.

### Slots

`X`: **matrix** of pseudo-observations based on which the empirical copula is constructed.  
`smoothing`: **character** string determining the smoothing method.  
`offset`: **numeric** giving the shift in the normalizing factor for computing the empirical copula.  
`ties.method`: a string indicating `rank()`'s ties method for computing the empirical copula.

### See Also

The class constructor are `empCopula()`, also for examples.

---

enacopula      *Estimation Procedures for (Nested) Archimedean Copulas*

---

### Description

A set of ten different estimators, currently for one-parameter Archimedean copulas, of possibly quite high dimensions.

**Usage**

```
enacopula(u, cop,
          method = c("mle", "smle", "dmle",
                    "mde.chisq.CvM", "mde.chisq.KS",
                    "mde.gamma.CvM", "mde.gamma.KS",
                    "tau.tau.mean", "tau.theta.mean", "beta"),
          n.MC = if (method == "smle") 10000 else 0,
          interval = initOpt(cop@copula@name),
          xargs = list(), ...)
```

**Arguments**

|          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| u        | $n \times d$ -matrix of (pseudo-)observations (each value in $[0, 1]$ ) from the copula to be estimated, where $n$ denotes the sample size and $d$ the dimension. Consider applying the function <code>pobs</code> first in order to obtain u.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| cop      | <code>outer_nacopula</code> to be estimated (currently only Archimedean copulas are provided).                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| method   | a <b>character</b> string specifying the estimation method to be used, which has to be one (or a unique abbreviation) of<br><p>"mle" maximum likelihood estimator (MLE) computed via <code>.emle</code>.</p> <p>"smle" simulated maximum likelihood estimator (SMLE) computed with the function <code>.emle</code>, where <code>n.MC</code> gives the Monte Carlo sample size.</p> <p>"dmle" MLE based on the diagonal (DMLE); see <code>edmle</code>.</p> <p>"mde.chisq.CvM" minimum distance estimator based on the chisq distribution and Cramér-von Mises distance; see <code>emde</code>.</p> <p>"mde.chisq.KS" minimum distance estimation based on the chisq distribution and Kolmogorov-Smirnov distance; see <code>emde</code>.</p> <p>"mde.gamma.CvM" minimum distance estimation based on the Erlang distribution and Cramér-von Mises distance; see <code>emde</code>.</p> <p>"mde.gamma.KS" minimum distance estimation based on the Erlang distribution and Kolmogorov-Smirnov distance; see <code>emde</code>.</p> <p>"tau.tau.mean" averaged pairwise Kendall's tau estimator</p> <p>"tau.theta.mean" average of pairwise Kendall's tau estimators</p> <p>"beta" multivariate Blomqvist's beta estimator</p> |
| n.MC     | only for <code>method = "smle"</code> : <b>integer</b> , sample size for simulated maximum likelihood estimation.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| interval | bivariate vector denoting the interval where optimization takes place. The default is computed as described in Hofert et al. (2012). Used for all methods except "tau.tau.mean" and "tau.theta.mean".                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| xargs    | list of additional arguments for the chosen estimation method.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| ...      | additional arguments passed to <code>optimize</code> .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |

## Details

[enacopula](#) serves as a wrapper for the different implemented estimators and provides a uniform framework to utilize them. For more information, see the single estimators as given in the section ‘See Also’.

Note that Hofert, Mächler, and McNeil (2013) compared these estimators. Their findings include a rather poor performance and numerically challenging problems of some of these estimators. In particular, the estimators obtained by `method="mde.gamma.CVM"`, `method="mde.gamma.KS"`, `method="tau.theta.mean"`, and `method="beta"` should be used with care (or not at all). Overall, MLE performed best (by far).

## Value

the estimated parameter,  $\hat{\theta}$ , that is, currently a number as only one-parameter Archimedean copulas are considered.

## References

Hofert, M., Mächler, M., and McNeil, A. J. (2012). Likelihood inference for Archimedean copulas in high dimensions under known margins. *Journal of Multivariate Analysis* **110**, 133–150.

Hofert, M., Mächler, M., and McNeil, A. J. (2013). Archimedean Copulas in High Dimensions: Estimators and Numerical Challenges Motivated by Financial Applications. *Journal de la Société Française de Statistique* **154**(1), 25–63.

## See Also

[emle](#) which returns an object of "mle" providing useful methods not available for other estimators. [demo](#)(opC-demo) and [vignette](#)("GIG", package="copula") for examples of two-parameter families. [edmle](#) for the diagonal maximum likelihood estimator. [emde](#) for the minimum distance estimators. [etau](#) for the estimators based on Kendall’s tau. [ebeta](#) for the estimator based on Blomqvist’s beta.

## Examples

```
tau <- 0.25
(theta <- copGumbel@iTau(tau)) # 4/3
d <- 12
(cop <- onacopulaL("Gumbel", list(theta,1:d)))

set.seed(1)
n <- 100
U <- rnacopula(n, cop)

meths <- eval(formals(enacopula)$method)

fun <- function(meth, u, cop, theta) {
  run.time <- system.time(val <- enacopula(u, cop=cop, method=meth))
  list(value=val, error=val-theta, utime.ms=1000*run.time[[1]])
}
t(res <- sapply(meths, fun, u=U, cop=cop, theta=theta))
```

estim.misc

*Various Estimators for (Nested) Archimedean Copulas***Description**

Various Estimators for (Nested) Archimedean Copulas, namely,

**ebeta** Method-of-moments-like estimator based on (a multivariate version of) Blomqvist's beta.

**edmle** Maximum likelihood estimator based on the diagonal of a (nested) Archimedean copula.

**etau** Method-of-moments-like estimators based on (bivariate) Kendall's tau.

**Usage**

```
ebeta(u, cop, interval = initOpt(cop@copula@name), ...)
edmle(u, cop, interval = initOpt(cop@copula@name), warn=TRUE, ...)
etau(u, cop, method = c("tau.mean", "theta.mean"), warn=TRUE, ...)
```

**Arguments**

|          |                                                                                                                                                                                                                                                                                                                        |
|----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| u        | $n \times d$ -matrix of (pseudo-)observations (each value in $[0, 1]$ ) from the copula, where $n$ denotes the sample size and $d$ the dimension.                                                                                                                                                                      |
| cop      | <a href="#">outer_nacopula</a> to be estimated (currently only Archimedean copulas are provided).                                                                                                                                                                                                                      |
| interval | bivariate vector denoting the interval where optimization takes place. The default is computed as described in Hofert et al. (2013).                                                                                                                                                                                   |
| method   | a character string specifying the method (only for etau), which has to be one (or a unique abbreviation) of<br>"tau.mean" method-of-moments-like estimator based on the average of pairwise sample versions of Kendall's tau;<br>"theta.mean" average of the method-of-moments-like Kendall's tau estimators.          |
| warn     | logical indicating if warnings are printed:<br>edmle() for the family of "Gumbel" if the diagonal maximum-likelihood estimator is smaller than 1.<br>etau() for the family of "AMH" if tau is outside $[0, 1/3]$ and in general if at least one of the computed pairwise sample versions of Kendall's tau is negative. |
| ...      | additional arguments passed to <a href="#">corKendall</a> (for etau, but see 'Details'), to <a href="#">optimize</a> (for edmle), or to <a href="#">safeUroot</a> (for ebeta).                                                                                                                                         |

## Details

For `ebeta`, the parameter is estimated with a method-of-moments-like procedure such that the population version of the multivariate Blomqvist's beta matches its sample version.

Note that the copula diagonal is a distribution function and the maximum of all components of a random vector following the copula is distributed according to this distribution function. For `edmle`, the parameter is estimated via maximum-likelihood estimation based on the diagonal.

For `etau`, `corKendall(u, ...)` is used and if there are no NAs in `u`, by default (if no additional arguments are provided), `corKendall()` calls the  $O(n \log(n))$  fast `cor.fk()` from package **pcaPP** instead of the  $O(n^2)$  `cor(*, method="kendall")`. Conversely, when `u` has NAs, by default, `corKendall(u, ...)` will use `cor(u, method="kendall", use="pairwise")` such that `etau(u, *)` will work. Furthermore, `method="tau.mean"` means that the average of sample versions of Kendall's tau are computed first and then the parameter is determined such that the population version of Kendall's tau matches this average (if possible); the `method="theta.mean"` stands for first computing all pairwise Kendall's tau estimators and then returning the mean of these estimators.

For more details, see Hofert et al. (2013).

Note that these estimators should be used with care; see the performance results in Hofert et al. (2013). In particular, `etau` should be used with the (default) method `"tau.mean"` since `"theta.mean"` is both slower and more prone to errors.

## Value

`ebeta` the return value of `safeUroot` (that is, typically almost the same as the value of `uniroot`) giving the Blomqvist beta estimator.

`edmle` `list` as returned by `optimize`, including the diagonal maximum likelihood estimator.

`etau` method-of-moments-like estimator based on Kendall's tau for the chosen method.

## References

Hofert, M., Mächler, M., and McNeil, A. J. (2013). Archimedean Copulas in High Dimensions: Estimators and Numerical Challenges Motivated by Financial Applications. *Journal de la Société Française de Statistique* **154**(1), 25–63.

## See Also

`corKendall()`.

The more sophisticated estimators `emle` (Maximum Likelihood) and `emde` (Minimum Distance). `enacopula` (wrapper for different estimators).

## Examples

```
tau <- 0.25
(theta <- copGumbel@iTau(tau)) # 4/3 = 1.333..
d <- 20
(cop <- onacopulaL("Gumbel", list(theta,1:d)))

set.seed(1)
n <- 200
```



```

U <- rncopula(n, cop)

system.time(theta.hat.beta <- ebeta(U, cop=cop))
theta.hat.beta$root

system.time(theta.hat.dmle <- edmle(U, cop=cop))
theta.hat.dmle$minimum

system.time(theta.hat.etau <- etau(U, cop=cop, method="tau.mean"))
theta.hat.etau

system.time(theta.hat.etau. <- etau(U, cop=cop, method="theta.mean"))
theta.hat.etau.

## etau() in the case of missing values (NA's)
## -----
##' @title add Missing Values completely at random
##' @param x matrix or vector
##' @param prob desired probability ("fraction") of missing values (\code{\link{NA}}s).
##' @return x[] with some (100*prob percent) entries replaced by \code{\link{NA}}s.
addNAs <- function(x, prob) {
  np <- length(x)
  x[sample.int(np, prob*np)] <- NA
  x
}

## UM[] := U[] with 5% missing values
set.seed(7); UM <- addNAs(U, p = 0.05)
mean(is.na(UM)) # 0.05
## This error if x has NA's was happening for etau(UM, cop=cop)
## before copula version 0.999-17 (June 2017) :
try(eM <- etau(UM, cop=cop, use = "everything"))
# --> Error ... NA/NaN/Inf in foreign function call
## The new default:
eM0 <- etau(UM, cop=cop, use = "pairwise")
eM1 <- etau(UM, cop=cop, use = "complete")
## use = "complete" is really equivalent to dropping all obs. with with missing values:
stopifnot(all.equal(eM1, etau(na.omit(UM), cop=cop), tol = 1e-15))
## but use = "pairwise" --> cor(*, use = "pairwise") is much better:
rbind(etau.U = theta.hat.etau, etau.UM.pairwise = eM0, etau.UM.complete = eM1)

```

**Description**

Constructs an extreme-value copula class object with its corresponding parameter.

**Usage**

```

evCopula(family, param, dim = 2, ...)
galambosCopula(param)
huslerReissCopula(param)
tawnCopula(param)
tevCopula(param, df = 4, df.fixed = FALSE)

```

**Arguments**

|          |                                                                                                                                                                                                         |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| family   | a character string specifying the family of an extreme-value copula. Currently implemented are "galambos", "gumbel", "huslerReiss", "tawn", "tev".                                                      |
| param    | a numeric vector specifying the parameter values.                                                                                                                                                       |
| dim      | the dimension of the copula. Currently, only "gumbel" allows $\text{dim} > 2$ .                                                                                                                         |
| df       | a number specifying the degrees of freedom of the t extreme-value copula, <code>tevCopula()</code> .                                                                                                    |
| df.fixed | logical; if true, the degrees of freedom will never be considered as a parameter to be estimated; FALSE means that df will be estimated if the object is passed as argument to <code>fitCopula</code> . |
| ...      | currently nothing.                                                                                                                                                                                      |

**Value**

An object of class "gumbelCopula", "galambosCopula", "huslerReissCopula", "tawnCopula", or "tevCopula".

**Note**

The Gumbel copula is both an Archimedean and an extreme-value copula, with principal documentation on `gumbelCopula` (or `archmCopula`).

**See Also**

`ellipCopula`, `archmCopula`, `gofEVCopula`, `An`.

**Examples**

```

## Gumbel is both
stopifnot(identical( evCopula("gumbel"), gumbelCopula()),
           identical(archmCopula("gumbel"), gumbelCopula()))

## For a given degree of dependence these copulas are strikingly similar :

tau <- 1/3

gumbel.cop    <- gumbelCopula    (iTau(gumbelCopula(),    tau))
galambos.cop  <- galambosCopula  (iTau(galambosCopula(), tau))
huslerReiss.cop <- huslerReissCopula(iTau(huslerReissCopula(), tau))
tawn.cop      <- tawnCopula      (iTau(tawnCopula(),      tau))
tev.cop       <- tevCopula       (iTau(tevCopula(),       tau))

```

```

curve(A(gumbel.cop, x), 0, 1, ylab = "A(<cop>( iTau(<cop>(), tau)), x)",
      main = paste("A(x) for five Extreme Value cop. w/ tau =", format(tau)))
curve(A(galambos.cop, x), lty=2, add=TRUE)
curve(A(huslerReiss.cop, x), lty=3, add=TRUE)
curve(A(tawn.cop, x), lty=4, add=TRUE)
curve(A(tev.cop, x), lty=5, col=2, add=TRUE)# very close to Gumbel

## And look at the differences
curve(A(gumbel.cop, x) - A(tawn.cop, x), ylim = c(-1,1)*0.005,
      ylab = '', main = "A(<Gumbel>, x) - A(<EV-Cop.>, x)")
abline(h=0, lty=2)
curve(A(gumbel.cop, x) - A(galambos.cop, x), add=TRUE, col=2)
curve(A(gumbel.cop, x) - A(huslerReiss.cop, x), add=TRUE, col=3)
curve(A(gumbel.cop, x) - A(tev.cop, x), add=TRUE, col=4, lwd=2)

## the t-EV-copula has always positive tau :
curve(vapply(x, function(x) tau(tevCopula(x)), 0.), -1, 1,
      n=257, ylim=0:1, xlab=quote(rho),ylab=quote(tau),
      main= quote(tau( tevCopula(rho) )), col = 2, lwd = 2)
rect(-1,0,1,1, lty = 2, border = adjustcolor("black", 0.5))

```

---

evCopula-class

*Classes Representing Extreme-Value Copulas*


---

## Description

Class `evCopula` is the virtual (mother) class of all extreme-value copulas. There currently are five subclasses, "`galambosCopula`", "`huslerReissCopula`", "`tawnCopula`", "`tevCopula`", and "`gumbelCopula`", the latter of which is also an Archimedean copula, see the page for class "`archmCopula`".

## Objects from the Class

`evCopula` is a virtual class: No objects may be created from it. Objects of class "`galambosCopula`" etc, can be created by calls of the form `new("galambosCopula", ...)`, but typically rather by `galambosCopula()`, etc, see there.

## Slots

All slots are inherited from the mother class "`copula`", see there.

## Methods

**dCopula** signature(`copula = "galambosCopula"`): ...  
**pCopula** signature(`copula = "galambosCopula"`): ...  
**rCopula** signature(`copula = "galambosCopula"`): ...  
**dCopula** signature(`copula = "huslerReissCopula"`): ...  
**pCopula** signature(`copula = "huslerReissCopula"`): ...  
**rCopula** signature(`copula = "huslerReissCopula"`): ...

**Extends**

Class "evCopula" extends class "[copula](#)" directly. Classes "[galambosCopula](#)", "[huslerReissCopula](#)", "[tawnCopula](#)", and "[tevCopula](#)" extend class "evCopula" directly.

**Note**

Objects of class "[gumbelCopula](#)" are also of class "[archmCopula](#)".

**See Also**

[evCopula](#), [evTestC](#), [evTestK](#), [gofEVCopula](#), [copula-class](#).

---

 evTestA

*Bivariate Test of Extreme-Value Dependence Based on Pickands' Dependence Function*

---

**Description**

Test of bivariate extreme-value dependence based on the process comparing the empirical copula with a natural nonparametric estimator of the unknown copula derived under extreme-value dependence. The test statistics are defined in the third reference. Approximate p-values for the test statistics are obtained by means of a *multiplier* technique.

**Usage**

```
evTestA(x, N = 1000, derivatives = c("An", "Cn"),
        ties.method = eval(formals(rank)$ties.method),
        trace.lev = 0, report.err = FALSE)
```

**Arguments**

|             |                                                                                                                                                                             |
|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x           | a data matrix that will be transformed to pseudo-observations.                                                                                                              |
| N           | number of multiplier iterations to be used to simulate realizations of the test statistic under the null hypothesis.                                                        |
| derivatives | string specifying how the derivatives of the unknown copula are estimated, either "An" or "Cn". The former gives better results for samples smaller than 400 but is slower. |
| ties.method | <a href="#">character</a> string specifying how ranks should be computed if there are ties in any of the coordinate samples of x; passed to <a href="#">pobs</a> .          |
| trace.lev   | integer indicating the level of diagnostic tracing to be printed to the console (from low-level algorithm).                                                                 |
| report.err  | <a href="#">logical</a> indicating if numerical integration errors should be reported in a summary way.                                                                     |

**Details**

More details are available in the third reference. See also Genest and Segers (2009) and Remillard and Scaillet (2009).

**Value**

An object of `class` `htest` which is a list, some of the components of which are

|                        |                                    |
|------------------------|------------------------------------|
| <code>statistic</code> | value of the test statistic.       |
| <code>p.value</code>   | corresponding approximate p-value. |

**Note**

This test was derived under the assumption of continuous margins, which implies that ties occur with probability zero. The presence of ties in the data might substantially affect the approximate p-value.

**References**

- Genest, C. and Segers, J. (2009). Rank-based inference for bivariate extreme-value copulas. *Annals of Statistics*, 37, pages 2990-3022.
- Rémillard, B. and Scaillet, O. (2009). Testing for equality between two copulas. *Journal of Multivariate Analysis*, 100(3), pages 377-386.
- Kojadinovic, I. and Yan, J. (2010). Nonparametric rank-based tests of bivariate extreme-value dependence. *Journal of Multivariate Analysis* **101**, 2234–2249.

**See Also**

[evTestK](#), [evTestC](#), [evCopula](#), [gofEVCopula](#), [An](#).

**Examples**

```
## Do these data come from an extreme-value copula?
set.seed(63)
uG <- rCopula(100, gumbelCopula(3))
uC <- rCopula(100, claytonCopula(3))
## these two take 21 sec on nb-mm4 (Intel Core i7-5600U @ 2.60GHz):
evTestA(uG)
evTestA(uC) # significant even though Clayton is *NOT* an extreme value copula

## These are fast:
evTestA(uG, derivatives = "Cn")
evTestA(uC, derivatives = "Cn") # small p-value even though Clayton is *NOT* an EV copula.
```

---

 evTestC

*Large-sample Test of Multivariate Extreme-Value Dependence*


---

### Description

Test of multivariate extreme-value dependence based on the empirical copula and max-stability. The test statistics are defined in the second reference. Approximate p-values for the test statistics are obtained by means of a *multiplier* technique.

### Usage

```
evTestC(x, N = 1000)
```

### Arguments

|   |                                                                                                                      |
|---|----------------------------------------------------------------------------------------------------------------------|
| x | a data matrix that will be transformed to pseudo-observations.                                                       |
| N | number of multiplier iterations to be used to simulate realizations of the test statistic under the null hypothesis. |

### Details

More details are available in the second reference. See also Remillard and Scaillet (2009).

### Value

An object of `class` `hctest` which is a list, some of the components of which are

|           |                                    |
|-----------|------------------------------------|
| statistic | value of the test statistic.       |
| p.value   | corresponding approximate p-value. |

### Note

This test was derived under the assumption of continuous margins, which implies that ties occur with probability zero. The presence of ties in the data might substantially affect the approximate p-value.

### References

Rémillard, B. and Scaillet, O. (2009). Testing for equality between two copulas. *Journal of Multivariate Analysis*, 100(3), pages 377-386.

Kojadinovic, I., Segers, J., and Yan, J. (2011). Large-sample tests of extreme-value dependence for multivariate copulas. *The Canadian Journal of Statistics* **39**, 4, pages 703-720.

### See Also

[evTestK](#), [evTestA](#), [evCopula](#), [gofEVCopula](#), [An](#).

**Examples**

```
## Do these data come from an extreme-value copula?
evTestC(rCopula(200, gumbelCopula(3)))
evTestC(rCopula(200, claytonCopula(3)))

## Three-dimensional examples
evTestC(rCopula(200, gumbelCopula(3, dim=3)))
evTestC(rCopula(200, claytonCopula(3, dim=3)))
```

---

|         |                                                                                   |
|---------|-----------------------------------------------------------------------------------|
| evTestK | <i>Bivariate Test of Extreme-Value Dependence Based on Kendall's Distribution</i> |
|---------|-----------------------------------------------------------------------------------|

---

**Description**

Test of extreme-value dependence based on the bivariate probability integral transformation. The test statistic is defined in Ben Ghorbal, G. Nešlehová, and Genest (2009).

**Usage**

```
evTestK(x, method = c("fsample", "asymptotic", "jackknife"), ties = NA, N = 100)
```

**Arguments**

|        |                                                                                                                                                                                                                                 |
|--------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x      | a data matrix.                                                                                                                                                                                                                  |
| method | specifies the variance estimation method; can be either "fsample" (finite-sample, the default), "asymptotic" or "jackknife".                                                                                                    |
| ties   | logical; if TRUE, the original test is adapted to take the presence of ties in the coordinate samples of x into account; the default value of NA indicates that the presence/absence of ties will be checked for automatically. |
| N      | number of samples to be used to estimate a bias term if ties = TRUE.                                                                                                                                                            |

**Details**

The code for this test was generously provided by Johanna G. Nešlehová. More details are available in Appendix B of Ben Ghorbal, G. Nešlehová and Genest (2009).

**Value**

An object of `class` htest which is a list, some of the components of which are

|           |                              |
|-----------|------------------------------|
| statistic | value of the test statistic. |
| p.value   | corresponding p-value.       |

## References

Ghorbal, M. B., Genest, C., and G. Nešlehová, J. (2009) On the test of Ghoudi, Khoudraji, and Rivest for extreme-value dependence. *The Canadian Journal of Statistics* **37**, 1–9.

Kojadinovic, I. (2017). Some copula inference procedures adapted to the presence of ties. *Computational Statistics and Data Analysis* **112**, 24–41, <https://arxiv.org/abs/1609.05519>.

## See Also

[evTestC](#), [evTestA](#), [evCopula](#), [gofEVCopula](#), [An](#).

## Examples

```
set.seed(321)
## Do the data come from an extreme-value copula?
evTestK(Ug <- rCopula(200, gumbelCopula(3))) # not significant => yes, EV
      dim(Uc <- rCopula(200, claytonCopula(3)))
## Clayton:                tests are highly significant => no, not EV
(K1 <- evTestK(Uc))
(K2 <- evTestK(Uc, method = "asymptotic"))

system.time(print(K3 <- evTestK(Uc, method = "jackknife")))
## slower: 1.06 sec (2015 intel core i7)
```

---

exchEVTest

*Test of Exchangeability for Certain Bivariate Copulas*

---

## Description

Test for assessing the exchangeability of the underlying bivariate copula when it is either extreme-value or left-tail decreasing. The test uses the nonparametric estimators of the Pickands dependence function studied by Genest and Segers (2009).

The test statistic is defined in the second reference. An approximate p-value for the test statistic is obtained by means of a *multiplier* technique if there are no ties in the component series of the bivariate data, or by means of an appropriate bootstrap otherwise.

## Usage

```
exchEVTest(x, N = 1000, estimator = c("CFG", "Pickands"),
           ties = NA, ties.method = eval(formals(rank)$ties.method),
           m = 100, derivatives = c("Cn", "An"))
```



**Arguments**

|             |                                                                                                                                                                                                                                                                                                     |
|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x           | a data matrix that will be transformed to pseudo-observations.                                                                                                                                                                                                                                      |
| N           | number of multiplier or bootstrap iterations to be used to simulate realizations of the test statistic under the null hypothesis.                                                                                                                                                                   |
| estimator   | string specifying which nonparametric estimator of the Pickands dependence function $A()$ to use; can be either "CFG" or "Pickands"; see Genest and Segers (2009).                                                                                                                                  |
| ties        | logical; if FALSE, approximate p-values are computed by means of a multiplier bootstrap; if TRUE, a bootstrap adapted to the presence of ties in any of the coordinate samples of x is used; the default value of NA indicates that the presence/absence of ties will be checked for automatically. |
| ties.method | string specifying how ranks should be computed if there are ties in any of the coordinate samples of x; passed to <code>pobs</code> .                                                                                                                                                               |
| derivatives | a string specifying how the derivatives of the unknown copula are estimated; can be either "An" or "Cn". The former should be used under the assumption of extreme-value dependence. The latter is faster; see the second reference.                                                                |
| m           | integer specifying the size of the integration grid for the statistic.                                                                                                                                                                                                                              |

**Details**

More details are available in the references.

**Value**

An object of class `htest` which is a list, some of the components of which are

|           |                                    |
|-----------|------------------------------------|
| statistic | value of the test statistic.       |
| pvalue    | corresponding approximate p-value. |

**References**

Genest, C. and Segers, J. (2009) Rank-based inference for bivariate extreme-value copulas. *Annals of Statistics* **37**, 2990–3022.

Kojadinovic, I. and Yan, J. (2012) A nonparametric test of exchangeability for extreme-value and left-tail decreasing bivariate copulas. *The Scandinavian Journal of Statistics* **39:3**, 480–496.

Kojadinovic, I. (2017). Some copula inference procedures adapted to the presence of ties. *Computational Statistics and Data Analysis* **112**, 24–41, <https://arxiv.org/abs/1609.05519>.

**See Also**

`exchTest`, `radSymTest`, `gofCopula`.

## Examples

```
## Data from an exchangeable left-tail decreasing copulas
exchEVTTest(rCopula(200, gumbelCopula(3)))
exchEVTTest(rCopula(200, claytonCopula(3)))

## An asymmetric Khoudraji-Gumbel-Hougaard copula
kc <- khoudrajiCopula(copula1 = indepCopula(),
                    copula2 = gumbelCopula(4),
                    shapes = c(0.4, 0.95))
exchEVTTest(rCopula(200, kc))
```

---

 exchTest

*Test of Exchangeability for a Bivariate Copula*


---

## Description

Test for assessing the exchangeability of the underlying bivariate copula based on the empirical copula. The test statistics are defined in the first two references. Approximate p-values for the test statistics are obtained by means of a *multiplier* technique if there are no ties in the component series of the bivariate data, or by means of an appropriate bootstrap otherwise.

## Usage

```
exchTest(x, N = 1000, ties = NA,
        ties.method = eval(formals(rank)$ties.method), m = 0)
```

## Arguments

|             |                                                                                                                                                                                                                                                                                                     |
|-------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x           | a data matrix that will be transformed to pseudo-observations.                                                                                                                                                                                                                                      |
| N           | number of multiplier or bootstrap iterations to be used to simulate realizations of the test statistic under the null hypothesis.                                                                                                                                                                   |
| ties        | logical; if FALSE, approximate p-values are computed by means of a multiplier bootstrap; if TRUE, a bootstrap adapted to the presence of ties in any of the coordinate samples of x is used; the default value of NA indicates that the presence/absence of ties will be checked for automatically. |
| ties.method | string specifying how ranks should be computed if there are ties in any of the coordinate samples of x; passed to <a href="#">pobs</a> .                                                                                                                                                            |
| m           | if m=0, integration in the Cramér–von Mises statistic is carried out with respect to the empirical copula; if m > 0, integration is carried out with respect to the Lebesgue measure and m specifies the size of the integration grid.                                                              |

## Details

More details are available in the references.

**Value**

An object of `class` `htest` which is a list, some of the components of which are

`statistic`      value of the test statistic.  
`p.value`        corresponding approximate p-value.

**References**

Genest, C., G. Nešlehová, J. and Quessy, J.-F. (2012). Tests of symmetry for bivariate copulas. *Annals of the Institute of Statistical Mathematics* **64**, 811–834.

Kojadinovic, I. and Yan, J. (2012). A nonparametric test of exchangeability for extreme-value and left-tail decreasing bivariate copulas. *The Scandinavian Journal of Statistics* **39:3**, 480–496.

Kojadinovic, I. (2017). Some copula inference procedures adapted to the presence of ties. *Computational Statistics and Data Analysis* **112**, 24–41, <https://arxiv.org/abs/1609.05519>.

**See Also**

[radSymTest](#), [exchEVTTest](#), [gofCopula](#).

**Examples**

```
## Data from an exchangeable copulas
exchTest(rCopula(200, gumbelCopula(3)))
exchTest(rCopula(200, claytonCopula(3)))

## An asymmetric Khoudraji-Clayton copula
kc <- khoudrajiCopula(copula1 = indepCopula(),
                     copula2 = claytonCopula(6),
                     shapes = c(0.4, 0.95))
exchTest(rCopula(200, kc))
```

---

fgmCopula

---

*Construction of a fgmCopula Class Object*


---

**Description**

Constructs a multivariate multiparameter Farlie-Gumbel-Morgenstern copula class object with its corresponding parameters and dimension.

**Usage**

```
fgmCopula(param, dim = 2)
```

**Arguments**

`param`            a numeric vector specifying the parameter values.  
`dim`                the dimension of the copula.

**Value**

A Farlie-Gumbel-Morgenstern copula object of class "[fgmCopula](#)".

**Note**

Note that a  $d$ -dimensional `fgmCopula` must have  $\text{npar} = 2^d - d - 1$  parameters. The verification of the validity of the parameter values is of high complexity and may not work for high dimensional copulas.

The random number generation needs to be properly tested, especially for dimensions higher than 2.

**References**

Nelsen, R. B. (2006), *An introduction to Copulas*, Springer, New York.

**See Also**

[Copula](#), [copula-class](#), [fitCopula](#).

**Examples**

```
## length(param) = #{parameters} for d-dimensional FGM copula:
d <- 2:10; rbind(d, npar = 2^d - d - 1)
## d      2    3    4    5    6    7    8    9   10
## npar   1    4   11   26   57  120  247  502 1013

## a bivariate example
fgm.cop <- fgmCopula(1)
x <- rCopula(1000, fgm.cop)
cor(x, method = "kendall")
tau(fgm.cop)
cor(x, method = "spearman")
rho(fgm.cop)
persp (fgm.cop, dCopula)
contour(fgm.cop, dCopula)

## a trivariate example with wrong parameter values
try(
  fgm2.cop <- fgmCopula(c(1,1,1,1), dim = 3)
) # Error: "Bad vector of parameters"

## a trivariate example with satisfactory parameter values
fgm2.cop <- fgmCopula(c(.2,-.2,-.4,.6), dim = 3)
fgm2.cop
```

---

|                 |                                                                                          |
|-----------------|------------------------------------------------------------------------------------------|
| fgmCopula-class | <i>Class "fgmCopula" - Multivariate Multiparameter Farlie-Gumbel-Morgenstern Copulas</i> |
|-----------------|------------------------------------------------------------------------------------------|

---

### Description

The class of multivariate multiparameter Farlie-Gumbel-Morgenstern copulas are typically created via `fgmCopula(...)`.

### Objects from the Class

Objects are typically created by `fgmCopula(...)`, or more low-level by (careful) calls to `new("fgmCopula", ...)`.

### Slots

**exprdist:** Object of class "expression", expressions for the cdf and pdf of the copula. These expressions are used in function `pCopula()` and `dCopula()`.

**subsets.char:** Object of class "character", containing the subsets of integers used for naming the parameters.

**dimension:** Object of class "numeric", the dimension of the copula.

**parameters:** Object of class "numeric", parameter values.

**param.names:** Object of class "character", parameter names.

**param.lowbnd:** Object of class "numeric", parameter lower bound.

**param.upbnd:** Object of class "numeric", parameter upper bound.

**fullname:** Object of class "character", family names of the copula (deprecated).

### Methods

**dCopula** signature(copula = "fgmCopula"): ...

**pCopula** signature(copula = "fgmCopula"): ...

**rCopula** signature(copula = "fgmCopula"): ...

### Extends

Class "fgmCopula" extends class "`copula`" directly.

### Note

The verification of the validity of the parameter values is of high complexity and may not work for high dimensional copulas.

The random number generation needs to be properly tested, especially for dimensions higher than 2.

**References**

Nelsen, R. B. (2006), *An introduction to Copulas*, Springer, New York.

**See Also**

[copula-class](#); to create such objects, use [fgmCopula\(\)](#); see there, also for examples.

---

 fhCopula

---

*Construction of Fréchet-Hoeffding Bound Copula Objects*


---

**Description**

Constructs the Fréchet-Hoeffding lower and upper bound copulas aka  $W$  and  $M$ .

**Usage**

```
fhCopula(family = c("upper", "lower"), dim = 2L)
```

```
lowfhCopula(dim = 2L)
```

```
upfhCopula(dim = 2L)
```

**Arguments**

|        |                                                                                                                   |
|--------|-------------------------------------------------------------------------------------------------------------------|
| family | a character string specifying the Fréchet-Hoeffding bound copula.                                                 |
| dim    | the dimension of the copula; note that the lower Fréchet-Hoeffding bound is only available in the bivariate case. |

**Value**

A copula object of class "[lowfhCopula](#)" or "[upfhCopula](#)".

**Note**

[fhCopula\(\)](#) is a wrapper for [lowfhCopula\(\)](#) and [upfhCopula\(\)](#).

The [dCopula\(\)](#) method will simply return an error for these copulas (as their density does not exist). Also, since the Fréchet-Hoeffding bound copulas are not parametric, certain methods available for parametric copulas are not available.

**Examples**

```
## Lower bound W : -----

try(W <- lowfhCopula(dim = 3)) # lower bound is *not* a copula for dim > 2
W <- lowfhCopula()
wireframe2(W, FUN = pCopula)
plot(W, n=100) # perfect anti-correlation ( rho = tau = -1 )
```

```
## Upper bound M : -----
wireframe2(upfhCopula(dim = 2), pCopula)
M <- upfhCopula(dim = 3)
set.seed(271)
splom2(M, n = 100) # "random" data: all perfectly correlated
```

---

|                |                                                            |
|----------------|------------------------------------------------------------|
| fhCopula-class | <i>Class "fhCopula" of Fréchet-Hoeffding Bound Copulas</i> |
|----------------|------------------------------------------------------------|

---

### Description

Fréchet-Hoeffding bound copula class.

### Objects from the Class

Created by calls of the form `new("fhCopula", ...)` or rather typically by `fhCopula()`, `lowfhCopula()`, or `upfhCopula()`. Actual (sub) classes are the lower and upper Fréchet-Hoeffding bound copulas `lowfhCopula()` ( $W$ ), and `upfhCopula()` ( $M$ ).

### Slots

**dimension:** inherited from super class `"dimCopula"`.

**exprdist:** an [expression](#) of length two, named `"cdf"` with the R expression of the CDF, and `"pdf"` which is empty as the PDF does not exist (everywhere).

### See Also

[ellipCopula](#), [archmCopula](#), [evCopula](#).

The class constructors are `fhCopula()`, `lowfhCopula()`, and `upfhCopula()`. See there for examples.

---

|           |                                                              |
|-----------|--------------------------------------------------------------|
| fitCopula | <i>Fitting Copulas to Data – Copula Parameter Estimation</i> |
|-----------|--------------------------------------------------------------|

---

### Description

Parameter estimation of copulas, i.e., fitting of a copula model to multivariate (possibly “pseudo”) observations.

**Usage**

```

loglikCopula(param = getTheta(copula), u, copula,
             error = c("-Inf", "warn-Inf", "let-it-be"))

loglikCopulaMany(pList, u, copula)

## Generic [and "rotCopula" method] : %- ../R/fitCopula.R
fitCopula(copula, data, ...)
## S4 method for signature 'parCopula'
fitCopula(copula, data,
          method = c("mpl", "ml", "itau", "irho", "itau.mpl"),
          posDef = is(copula, "ellipCopula"),
          start = NULL, lower = NULL, upper = NULL,
          optim.method = optimMeth(copula, method, dim = d),
          optim.control = list(maxit=1000),
          estimate.variance = NA, hideWarnings = FALSE, ...)

optimMeth(copula, method, dim)

```

**Arguments**

|        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|--------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| param  | vector of <i>free</i> (see <code>isFree()</code> and <code>getTheta()</code> ) parameter values.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| pList  | a <b>list</b> of <i>free</i> parameter vectors (as param above). In the 1D case, <code>length(param) == 1</code> , may also be a numeric vector.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            |
| u      | $n \times d$ -matrix of (pseudo-)observations in $[0, 1]^d$ for computing the copula log-likelihood, where $n$ denotes the sample size and $d$ the dimension. Consider applying the function <code>pobs()</code> first in order to obtain such data.                                                                                                                                                                                                                                                                                                                                                                                                                        |
| data   | as $u$ , an $n \times d$ -matrix of data. For method being "mpl", "ml" or "itau.mpl", this has to be data in $[0, 1]^d$ . For method being "itau" or "irho", it can either be data in $[0, 1]^d$ or in the whole $d$ -dimensional space.                                                                                                                                                                                                                                                                                                                                                                                                                                    |
| copula | a " <b>copula</b> " object.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| error  | (for <code>loglikCopula()</code> ): a <b>character</b> string specifying how errors in the underlying <code>dCopula()</code> calls should be handled:<br>"-Inf": the value of the log likelihood should silently be set to -Inf.<br>"warn-Inf": signal a <b>warning</b> about the error and set the value to -Inf.<br>"let-it-be": the error is signalled and hence the likelihood computation fails.                                                                                                                                                                                                                                                                       |
| method | a <b>character</b> string specifying the copula parameter estimator used. This can be one of:<br><b>"mpl"</b> Maximum pseudo-likelihood estimator (based on "pseudo-observations" in $[0, 1]^d$ , typical obtained via <code>pobs()</code> ).<br><b>"ml"</b> As "mpl" just with a different variance estimator. For this to be correct (thus giving the true MLE), data are assumed to be observations from the true underlying copula whose parameter is to be estimated.<br><b>"itau"</b> Inversion of Kendall's tau estimator. data can be either in $[0, 1]^d$ (true or pseudo-observations of the underlying copula to be estimated) or in the $d$ -dimensional space. |



|                                |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       |
|--------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                                | <b>"irho"</b> As "itau" just with Spearman's rho instead of Kendall's tau.                                                                                                                                                                                                                                                                                                                                                                                                                            |
|                                | <b>"itau.mpl"</b> This is the estimator of $t$ copula parameters suggested by Zeevi and Mashal (2002) based on the idea of inverting Kendall's tau for estimating the correlation matrix as introduced in a RiskLab report in 2001 later published as Embrechts et al. (2003); see also Demarta and McNeil (2005). The given data has to be in $[0, 1]^d$ (either true or pseudo-observations of the underlying copula to be estimated). Note that this method requires <code>dispstr = "un"</code> . |
| <code>posDef</code>            | a <b>logical</b> indicating whether a proper correlation matrix is computed.                                                                                                                                                                                                                                                                                                                                                                                                                          |
| <code>start</code>             | a <b>vector</b> of starting values for the parameter optimization via <code>optim()</code> .                                                                                                                                                                                                                                                                                                                                                                                                          |
| <code>lower, upper</code>      | Lower or upper parameter bounds for the optimization methods "Brent" or "L-BFGS-B".                                                                                                                                                                                                                                                                                                                                                                                                                   |
| <code>optim.control</code>     | a <b>list</b> of control parameters passed to <code>optim(*, control=optim.control)</code> .                                                                                                                                                                                                                                                                                                                                                                                                          |
| <code>optim.method</code>      | a character string specify the optimization method <i>or</i> a <b>function</b> which when called with arguments (copula, method, dim) will return such a character string, see <code>optim()</code> 's method; only used when <code>method = "mpl"</code> or <code>"ml"</code> .<br>The default has been changed (for <b>copula</b> 0.999-16, in Aug. 2016) from "BFGS" to the result of <code>optimMeth(copula, method, dim)</code> which is often "L-BFGS-B".                                       |
| <code>dim</code>               | integer, the data and copula dimension, $d \geq 2$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
| <code>estimate.variance</code> | a <b>logical</b> indicating whether the estimator's asymptotic variance is computed (if available for the given copula; the default NA computes it for the methods "itau" and "irho", cannot (yet) compute it for "itau.mpl" and only computes it for "mpl" or "ml" if the optimization converged).                                                                                                                                                                                                   |
| <code>hideWarnings</code>      | a <b>logical</b> , which, if <b>TRUE</b> , suppresses warnings from the involved likelihood maximization (typically when the likelihood is evaluated at invalid parameter values).                                                                                                                                                                                                                                                                                                                    |
| <code>...</code>               | additional arguments passed to method specific auxiliary functions, e.g., <code>traceOpt = TRUE</code> (or <code>traceOpt = 10</code> ) for tracing <code>optimize</code> (every 10-th function evaluation) for method "itau.mpl", and for "manual" tracing with method "ml" or "mpl" also showing parameter values (notably for <code>optim.method="Brent"</code> ), see the extra arguments of namespace-hidden function <code>fitCopula.ml()</code> .                                              |

## Details

The only difference between "mpl" and "ml" is in the variance-covariance estimate, *not* in the parameter ( $\theta$ ) estimates.

If method "mpl" in `fitCopula()` is used and if `start` is not assigned a value, estimates obtained from method "itau" are used as initial values in the optimization. Standard errors are computed as explained in Genest, Ghoudi and Rivest (1995); see also Kojadinovic and Yan (2010, Section 3). Their estimation requires the computation of certain partial derivatives of the (log) density. These have been implemented for six copula families thus far: the Clayton, Gumbel-Hougaard, Frank, Plackett, normal and  $t$  copula families. For other families, numerical differentiation based on `grad()` from package **numDeriv** is used (and a warning message is displayed).

In the multiparameter elliptical case and when the estimation is based on Kendall's tau or Spearman's rho, the estimated correlation matrix may not always be positive-definite. In that case, `nearPD(*, corr=TRUE)` (from **Matrix**) is applied to get a proper correlation matrix.

For normal and  $t$  copulas, `fitCopula(, method = "mpl")` and `fitCopula(, method = "ml")` maximize the log-likelihood based on **mvtnorm**'s `dmvnorm()` and `dmvt()`, respectively. The latter two functions set the respective densities to zero if the correlation matrices of the corresponding distributions are not positive definite. As such, the estimated correlation matrices will be positive definite.

If methods "itau" or "irho" are used in `fitCopula()`, an estimate of the asymptotic variance (if available for the copula under consideration) will be correctly computed only if the argument `data` consists of pseudo-observations (see `pobs()`).

Consider the  $t$  copula with `df.fixed=FALSE` (see `ellipCopula()`). In this case, the methods "itau" and "irho" cannot be used in `fitCopula()` as they cannot estimate the degrees of freedom parameter `df`. For the methods "mpl" and "itau.mpl" the asymptotic variance cannot be (fully) estimated (yet). For the methods "ml" and "mpl", when `start` is not specified, the starting value for `df` is set to `copula@df`, typically 4.

To implement the *Inference Functions for Margins* (IFM) method (see, e.g., Joe 2005), set `method="ml"` and note that data need to be parametric pseudo-observations obtained from *fitted* parametric marginal distribution functions. The returned large-sample variance will then underestimate the true variance (as the procedure cannot take into account the (unknown) estimation error for the margins).

The fitting procedures based on `optim()` generate warnings because invalid parameter values are tried during the optimization process. When the number of parameters is one and the parameter space is bounded, using `optim.method="Brent"` is likely to give less warnings. Furthermore, from experience, `optim.method="Nelder-Mead"` is sometimes a more robust alternative to `optim.method="BFGS"` or "L-BFGS-B".

There are methods for `vcov()`, `coef()`, `logLik()`, and `nobs()`.

## Value

`loglikCopula()` returns the copula log-likelihood evaluated at the parameter (vector) `param` given the data `u`.

`loglikCopulaMany()` returns a numeric vector of such log-likelihoods; it assumes consistent parameter values, corresponding to `loglikCopula()`'s `error = "let-it-be"`, for speed.

The return value of `fitCopula()` is an object of class "fitCopula" (inheriting from hidden class "fittedMV"), containing (among others!) the slots

**estimate** The parameter estimates.

**var.est** The large-sample (i.e., asymptotic) variance estimate of the parameter estimator unless `estimate.variance=FALSE` where it is `matrix(numeric(), 0, 0)` (to be distinguishable from cases when the covariance estimates failed partially).

**copula** The fitted copula object.

The `summary()` method for "fitCopula" objects returns an S3 "class" "summary.fitCopula", which is simply a list with components `method`, `loglik` and `convergence`, all three from the corresponding slots of the "fitCopula" objects, and `coefficients` (a matrix of estimated coefficients, standard errors,  $t$  values and  $p$ -values).

## References

- Genest, C. (1987). Frank's family of bivariate distributions. *Biometrika* **74**, 549–555.
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- Rousseeuw, P. and Molenberghs, G. (1993). Transformation of nonpositive semidefinite correlation matrices. *Communications in Statistics: Theory and Methods* **22**, 965–984.
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## See Also

[Copula](#), [fitMvdc](#) for fitting multivariate distributions *including* the margins, [gofCopula](#) for goodness-of-fit tests.

For maximum likelihood of (nested) Archimedean copulas, see [emle](#), etc.

## Examples

```
(Xtras <- copula::doExtras()) # determine whether examples will be extra (long)
n <- if(Xtras) 200 else 64 # sample size

## A Gumbel copula
set.seed(7) # for reproducibility
gumbel.cop <- gumbelCopula(3, dim=2)
x <- rCopula(n, gumbel.cop) # "true" observations (simulated)
u <- pobs(x) # pseudo-observations
## Inverting Kendall's tau
fit.tau <- fitCopula(gumbelCopula(), u, method="itau")
fit.tau
confint(fit.tau) # work fine !
confint(fit.tau, level = 0.98)
summary(fit.tau) # a bit more, notably "Std. Error"s
coef(fit.tau) # named vector
coef(fit.tau, SE = TRUE) # matrix

## Inverting Spearman's rho
fit.rho <- fitCopula(gumbelCopula(), u, method="irho")
summary(fit.rho)
```

```

## Maximum pseudo-likelihood
fit.mpl <- fitCopula(gumbelCopula(), u, method="mpl")
fit.mpl
## Maximum likelihood -- use 'x', not 'u' ! --
fit.ml <- fitCopula(gumbelCopula(), x, method="ml")
summary(fit.ml) # now prints a bit more than simple 'fit.ml'
## ... and what's the log likelihood (in two different ways):
(ll. <- logLik(fit.ml))
stopifnot(all.equal(as.numeric(ll.),
                    loglikCopula(coef(fit.ml), u=x, copula=gumbel.cop)))

## A Gauss/normal copula

## With multiple/*un*constrained parameters
set.seed(6) # for reproducibility
normal.cop <- normalCopula(c(0.6, 0.36, 0.6), dim=3, dispstr="un")
x <- rCopula(n, normal.cop) # "true" observations (simulated)
u <- pobs(x)                # pseudo-observations
## Inverting Kendall's tau
fit.tau <- fitCopula(normalCopula(dim=3, dispstr="un"), u, method="itau")
fit.tau
## Inverting Spearman's rho
fit.rho <- fitCopula(normalCopula(dim=3, dispstr="un"), u, method="irho")
fit.rho
## Maximum pseudo-likelihood
fit.mpl <- fitCopula(normalCopula(dim=3, dispstr="un"), u, method="mpl")
summary(fit.mpl)
coef(fit.mpl) # named vector
coef(fit.mpl, SE = TRUE) # the matrix, with SE
## Maximum likelihood (use 'x', not 'u' !)
fit.ml <- fitCopula(normalCopula(dim=3, dispstr="un"), x, method="ml", traceOpt=TRUE)
summary(fit.ml)
confint(fit.ml)
confint(fit.ml, level = 0.999) # clearly non-0

## Fix some of the parameters
param <- c(.6, .3, NA_real_)
attr(param, "fixed") <- c(TRUE, FALSE, FALSE)
ncp <- normalCopula(param = param, dim = 3, dispstr = "un")
fixedParam(ncp) <- c(TRUE, TRUE, FALSE)
## 'traceOpt = 5': showing every 5-th log likelihood evaluation:
summary(Fxf.mpl <- fitCopula(ncp, u, method = "mpl", traceOpt = 5))
Fxf.mpl@copula # reminding of the fixed param. values

## With dispstr = "toep" :
normal.cop.toep <- normalCopula(c(0, 0), dim=3, dispstr="toep")
## Inverting Kendall's tau
fit.tau <- fitCopula(normalCopula(dim=3, dispstr="toep"), u, method="itau")
fit.tau
## Inverting Spearman's rho
fit.rho <- fitCopula(normalCopula(dim=3, dispstr="toep"), u, method="irho")
summary(fit.rho)
## Maximum pseudo-likelihood

```

```

fit.mpl <- fitCopula(normalCopula(dim=3, dispstr="toep"), u, method="mpl")
fit.mpl
## Maximum likelihood (use 'x', not 'u' !)
fit.ml <- fitCopula(normalCopula(dim=3, dispstr="toep"), x, method="ml")
summary(fit.ml)

## With dispstr = "ar1"
normal.cop.ar1 <- normalCopula(c(0), dim=3, dispstr="ar1")
## Inverting Kendall's tau
summary(fit.tau <- fitCopula(normalCopula(dim=3, dispstr="ar1"), u, method="itau"))
## Inverting Spearman's rho
summary(fit.rho <- fitCopula(normalCopula(dim=3, dispstr="ar1"), u, method="irho"))
## Maximum pseudo-likelihood
summary(fit.mpl <- fitCopula(normalCopula(dim=3, dispstr="ar1"), u, method="mpl"))
## Maximum likelihood (use 'x', not 'u' !)
fit.ml <- fitCopula(normalCopula(dim=3, dispstr="ar1"), x, method="ml")
summary(fit.ml)

## A t copula with variable df (df.fixed=FALSE)
(tCop <- tCopula(c(0.2,0.4,0.6), dim=3, dispstr="un", df=5))
set.seed(101)
x <- rCopula(n, tCop) # "true" observations (simulated)
## Maximum likelihood (start = (rho[1:3], df))
summary(tc.ml <- fitCopula(tCopula(dim=3, dispstr="un"), x, method="ml",
                          start = c(0,0,0, 10)))
## Maximum pseudo-likelihood (the asymptotic variance cannot be estimated)
u <- pobs(x) # pseudo-observations
tc.mpl <- fitCopula(tCopula(dim=3, dispstr="un"),
                  u, method="mpl", estimate.variance=FALSE,
                  start= c(0,0,0, 10))

summary(tc.mpl)

```

---

fitCopula-class

*Classes of Fitted Multivariate Models: Copula, Mvdc*


---

## Description

Classes and summary methods related to copula model fitting.

## Objects from the Class

Objects can be created by calls to [fitCopula](#) or [fitMvdc](#), respectively or to their summary methods.

## Slots

The “mother class”, “fittedMV” has the slots

**estimate:** numeric, the estimated parameters.

**var.est:** numeric, variance matrix estimate of the parameter estimator. See note below.

loglik: numeric, log likelihood evaluated at the maximizer.  
 nsample: numeric, integer representing the sample size.  
 method: character, method of estimation.  
 fitting.stats: a `list`, currently containing the numeric convergence code from `optim`, the counts, message, and all the control arguments explicitly passed to `optim()`. Since `copula` version 1.0-1 also keeps information about parameter transformations, currently needed only for `mixCopula` fits with free weights.

In addition, the "fitCopula" class has a slot

copula: the *fitted* copula, of class "copula".

whereas the "fitMvdc" has

mvdc: the *fitted* distribution, of class "mvdc".

### Extends

Classes "fitCopula" and "fitMvdc" extend class "fittedMV", directly.

### Methods

**summary** signature(object = "fitMvdc"): ...

**summary** signature(object = "fitCopula"): ...

Further, there are S3 methods (class "fittedMV") for `coef()`, `vcov()` and `logLik()`, see `fitMvdc`.

### References

Genest, C., Ghoudi, K., and Rivest, L.-P. (1995). A semiparametric estimation procedure of dependence parameters in multivariate families of distributions. *Biometrika* **82**, 543–552.

---

fitLambda

*Non-parametric Estimators of the Matrix of Tail-Dependence Coefficients*

---

### Description

Computing non-parametric estimators of the (matrix of) tail-dependence coefficients.

### Usage

```
fitLambda(u, method = c("Schmid.Schmidt", "Schmidt.Stadtmueller", "t"),
  p = 1/sqrt(nrow(u)), lower.tail = TRUE, verbose = FALSE, ...)
```

**Arguments**

|            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
|------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| u          | $n \times d$ -matrix of (pseudo-)observations in $[0, 1]^d$ for estimating the (matrix of) tail-dependence coefficients.                                                                                                                                                                                                                                                                                                                                                                                                                                |
| method     | the method with which the tail-dependence coefficients are computed:<br>method = "Schmid.Schmidt": nonparametric estimator of Schmid and Schmidt (2007) (see also Jaworski et al. (2009, p. 231)) computed for all pairs (pairwise conditional Spearman's rho for $u \leq p$ ).<br>method = "Schmidt.Stadtmueller": nonparametric estimator of Schmidt and Stadtmueller (2006) computed for all pairs (pairwise empirical tail copula for $u \leq p$ ).<br>method = "t": fits pairwise $t$ copulas and returns the implied tail-dependence coefficient. |
| p          | (small) cut-off parameter in $[0, 1]$ below (for tail = "lower") or above (for tail = "upper") which the estimation takes place.                                                                                                                                                                                                                                                                                                                                                                                                                        |
| lower.tail | <b>logical</b> indicating whether the lower (the default) or upper tail-dependence coefficient is computed; in case of the latter, the lower tail dependence coefficient of the flipped data $1-u$ is computed.                                                                                                                                                                                                                                                                                                                                         |
| verbose    | a <b>logical</b> indicating whether a progress bar is displayed.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| ...        | additional arguments passed to the underlying functions (at the moment only to <code>optimize()</code> in case method = "t").                                                                                                                                                                                                                                                                                                                                                                                                                           |

**Details**

As seen in the examples, be careful using nonparametric estimators, they might not perform too well (depending on  $p$  and in general). After all, the notion of tail dependence is a limit, finite sample sizes may not be able to capture limits well.

**Value**

The matrix of pairwise coefficients of tail dependence; for method = "t" a **list** with components Lambda, the matrix of pairwise estimated correlation coefficients P and the matrix of pairwise estimated degrees of freedom Nu.

**References**

- Jaworski, P., Durante, F., Härdle, W. K., Rychlik, T. (2010). *Copula Theory and Its Applications* Springer, Lecture Notes in Statistics – Proceedings.
- Schmid, F., Schmidt, R. (2007). Multivariate conditional versions of Spearman's rho and related measures of tail dependence. *Journal of Multivariate Analysis* **98**, 1123–1140. doi:10.1016/j.jmva.2006.05.005
- Schmidt, R., Stadtmueller, U. (2006). Non-parametric Estimation of Tail Dependence. *Scandinavian Journal of Statistics* **33**(2), 307–335. doi:10.1111/j.14679469.2005.00483.x

**See Also**

`lambda()` computes the true (lower and upper) tail coefficients for a given copula.

**Examples**

```

n <- 10000 # sample size
p <- 0.01 # cut-off

## Bivariate case
d <- 2
cop <- claytonCopula(2, dim = d)
set.seed(271)
U <- rCopula(n, copula = cop) # generate observations (unrealistic)
(lam.true <- lambda(cop)) # true tail-dependence coefficients lambda
(lam.C <- c(lower = fitLambda(U, p = p)[2,1],
            upper = fitLambda(U, p = p, lower.tail = FALSE)[2,1])) # estimate lambdas
## => pretty good
U. <- pobs(U) # pseudo-observations (realistic)
(lam.C. <- c(lower = fitLambda(U., p = p)[2,1],
            upper = fitLambda(U., p = p, lower.tail = FALSE)[2,1])) # estimate lambdas
## => The pseudo-observations do have an effect...

## Higher-dimensional case
d <- 5
cop <- claytonCopula(2, dim = d)
set.seed(271)
U <- rCopula(n, copula = cop) # generate observations (unrealistic)
(lam.true <- lambda(cop)) # true tail-dependence coefficients lambda
(Lam.C <- list(lower = fitLambda(U, p = p),
              upper = fitLambda(U, p = p, lower.tail = FALSE))) # estimate Lambdas
## => Not too good
U. <- pobs(U) # pseudo-observations (realistic)
(Lam.C. <- list(lower = fitLambda(U., p = p),
              upper = fitLambda(U., p = p, lower.tail = FALSE))) # estimate Lambdas
## => Performance not too great here in either case

```

fitMvdc

*Estimation of Multivariate Models Defined via Copulas***Description**

Fitting copula-based multivariate distributions ("**mvdc**") to multivariate data, estimating both the marginal and the copula parameters.

If you assume (non parametric) margins, in other words, take the empirical distributions for all margins, you can use `fitCopula(*, pobs(x))` instead.

**Usage**

```

loglikMvdc(param, x, mvdc)
fitMvdc(data, mvdc, start, optim.control = list(), method = "BFGS",
        lower = -Inf, upper = Inf,
        estimate.variance = fit$convergence == 0, hideWarnings = TRUE)

```



```
## S3 method for class 'fittedMV'
coef(object, SE = FALSE, orig = TRUE, ...)
## S3 method for class 'fittedMV'
logLik(object, ...)
## S3 method for class 'fittedMV'
vcov(object, orig = TRUE, ...)
```

## Arguments

|                   |                                                                                                                                                                                                                                                                             |
|-------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| param             | a vector of parameter values. When specifying parameters for mvdc objects, the parameters must be ordered with the marginals first and the copula parameters last. When the mvdc object has marginsIdentical = TRUE, only the parameters of one marginal must be specified. |
| x                 | a data matrix.                                                                                                                                                                                                                                                              |
| mvdc              | a "mvdc" object.                                                                                                                                                                                                                                                            |
| data              | a data matrix.                                                                                                                                                                                                                                                              |
| start             | a vector of starting value for "param". See "param" above for ordering of this vector.                                                                                                                                                                                      |
| optim.control     | a list of controls to be passed to <a href="#">optim</a> .                                                                                                                                                                                                                  |
| method            | the method for <a href="#">optim</a> .                                                                                                                                                                                                                                      |
| lower, upper      | bounds on each parameter, passed to <a href="#">optim</a> , typically "box constraints" for method = "L-BFGS-B".                                                                                                                                                            |
| estimate.variance | logical; if true (as by default, if the optimization converges), the asymptotic variance is estimated.                                                                                                                                                                      |
| hideWarnings      | logical indicating if warning messages from likelihood maximization, e.g., from evaluating at invalid parameter values, should be suppressed (via <a href="#">suppressWarnings</a> ).                                                                                       |
| object            | an R object of class "fitMvdc".                                                                                                                                                                                                                                             |
| SE                | for the <a href="#">coef</a> method, a logical indicating if standard errors should be returned in addition to the estimated parameters (in a <a href="#">matrix</a> ). This is equivalent, but more efficient than, e.g., <a href="#">coef(summary(object))</a> .          |
| orig              | <a href="#">logical</a> , relevant currently only for <a href="#">mixCopula</a> fits with free weights. orig indicates if the weights should be shown in original scale (orig=TRUE) or in the transformed log- aka lambda-space.                                            |
| ...               | potentially further arguments to methods.                                                                                                                                                                                                                                   |

## Value

The return value [loglikMvdc\(\)](#) is the log likelihood evaluated for the given value of param.

The return value of [fitMvdc\(\)](#) is an object of class "fitMvdc" (see there), containing slots (among others!):

|          |                                                                                                                                             |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------|
| estimate | the estimate of the parameters.                                                                                                             |
| var.est  | large-sample (i.e., asymptotic) variance estimate of the parameter estimator (filled with NA if <a href="#">estimate.variance</a> = FALSE). |

`mvdc` the *fitted* multivariate distribution, see [mvdc](#).

The `summary()` method for "fitMvdc" objects returns a S3 "class" "summary.fitMvdc", simply a list with components `method`, `loglik`, and `convergence`, all three from corresponding slots of the "fitMvdc" objects, and

`coefficients` a matrix of estimated coefficients, standard errors, t values and p-values.

### Note

User-defined marginal distributions can be used as long as the "{dpq}" functions are defined. See `vignette("AR_Clayton", package="copula")`.

When covariates are available for marginal distributions or for the copula, one can construct the loglikelihood function and feed it to "optim" to estimate all the parameters.

Finally, note that some of the fitting functions generate error messages because invalid parameter values are tried during the optimization process (see [optim](#)). This should be rarer since 2013, notably for likelihood based methods (as the likelihood is now rather set to  $-\infty$  than giving an error).

Previously, `loglikMvdc()` had an argument `hideWarnings`; nowadays, do use `suppressWarnings(...)` if you are sure you do not want to see them.

### See Also

[mvdc](#) and [mvdc](#); further, [Copula](#), [fitCopula](#), [gofCopula](#).

For fitting univariate marginals, [fitdistr\(\)](#).

### Examples

```
G3 <- gumbelCopula(3, dim=2)
gMvd2 <- mvdc(G3, c("exp","exp"),
              param = list(list(rate=2), list(rate=4)))
## with identical margins:
gMvd.I <- mvdc(G3, "exp",
              param = list(rate=3), marginsIdentical=TRUE)

(Xtras <- copula::doExtras()) # determine whether examples will be extra (long)
n <- if(Xtras) 10000 else 200 # sample size (realistic vs short for example)

set.seed(11)
x <- rMvdc(n, gMvd2)
## Default:   hideWarnings = FALSE .. i.e. show warnings here
fit2 <- fitMvdc(x, gMvd2, start = c(1,1, 2))
fit2
confint(fit2)
summary(fit2) # slightly more
## The estimated, asymptotic var-cov matrix [was used for confint()]:
vcov(fit2)

## with even more output for the "identical margin" case:
fitI <- fitMvdc(x, gMvd.I, start = c(3, 2),
```

```

optim.control=list(trace= TRUE, REPORT= 2))
summary(fitI)
coef(fitI, SE = TRUE)
stopifnot(is.matrix(coef(fitI, SE = TRUE)),
          is.matrix(print( confint(fitI) )) )

## a wrong starting value can already be *the* problem:
f2 <- try(fitMvdc(x, gMvd.I, start = c(1, 1),
               optim.control=list(trace= TRUE, REPORT= 2)))
##--> Error in optim( ... ) : non-finite finite-difference value [2]

##=> "Solution" : Using a more robust (but slower) optim() method:
fI.2 <- fitMvdc(x, gMvd.I, start = c(1, 1), method = "Nelder",
               optim.control=list(trace= TRUE))
fI.2

```

---

fixParam

*Fix a Subset of a Copula Parameter Vector*


---

## Description

It is sometimes useful to keep fixed some components of a copula parameter vector whereas the others are “free” and will be estimated, e.g., by [fitCopula](#).

The first two functions set or modify the “fixedness”, whereas [isFree\(\)](#), [isFreeP\(\)](#) and [nParam\(\)](#) are utilities enquiring about the “fixedness” of the parameters (of a copula).

## Usage

```

fixParam(param, fixed = TRUE)
fixedParam(copula) <- value

isFreeP(param)
## S4 method for signature 'copula'
isFree(copula)
## and specific '*Copula' methods
## S4 method for signature 'copula'
nParam(copula, freeOnly = FALSE)
## and specific '*Copula' methods

```

## Arguments

|              |                                                                                                                        |
|--------------|------------------------------------------------------------------------------------------------------------------------|
| param        | numeric parameter vector                                                                                               |
| fixed, value | logical vector of the same length as param indicating for each component param[j] if it is (going to be) fixed or not. |
| copula       | a “ <a href="#">copula</a> ” object.                                                                                   |
| freeOnly     | logical (scalar) indicating if only free parameters should be counted or all.                                          |

**Value**

`fixParam(param)` returns a **numeric** vector with attribute "fixed" (a **logical**, either TRUE or vector of the same length as `param`) to indicate which components of `param` are to be held fixed or not.

`fixedParam<-`, a generic function, returns a "copula" object with a partly fixed parameter (slot), i.e., corresponding to `fixParam()` above.

**See Also**

`fitCopula` for fitting; t-copulas, `tCopula(*, df.fixed=TRUE)` now uses parameter fixing for "df".  
`setTheta()` for setting or *changing* the *non-fixed* parameter values.

**Examples**

```
nc1 <- normalCopula(dim = 3, fixParam(c(.6,.3,.2), c(TRUE, FALSE,FALSE)),
                    dispstr = "un")
nc1
nc13 <- nc12 <- nc1
fixedParam(nc12) <- c(TRUE, TRUE, FALSE) ; nc12
fixedParam(nc13) <- c(TRUE, FALSE, TRUE) ; nc13
set.seed(17); x <- rCopula(100, nc1)
summary(f.13 <- fitCopula(nc13, x, method = "ml"))
f.13@copula # 'rho.2' is estimated; the others kept fixed

## Setting to 'FALSE' (removes the "fixed" parts completely):
nc0 <- nc13; fixedParam(nc0) <- FALSE
nc0
stopifnot(is.null(attributes(nc0@parameters)))
```

---

 gasoil

*Daily Crude Oil and Natural Gas Prices from 2003 to 2006*


---

**Description**

Three years of daily prices (from July 2003 to July 2006) of crude oil and natural gas. These data should be very close to those analysed in Grégoire, Genest and Gendron (2008).

**Usage**

```
data(gasoil, package="copula")
```

**Format**

A data frame of 762 daily prices from 2003 to 2006.

`date` date (of class `Date`).

`oil` daily price of crude oil

`gas` daily price of natural gas

## References

Grégoire, V., Genest, C., and Gendron, M. (2008) Using copulas to model price dependence in energy markets. *Energy Risk* 5(5), 58–64.

## Examples

```
data(gasoil)
## Log Scaled Oil & Gas Prices :
lattice :: xyplot(oil + gas ~ date, data = gasoil, auto.key=TRUE,
                 type = c("l", "r"),
                 scales=list(y = list(log = TRUE), equispaced.log = FALSE))
```

---

generator

*Generator Functions for Archimedean and Extreme-Value Copulas*

---

## Description

Methods to evaluate the generator function, the inverse generator function, and derivatives of the inverse of the generator function for Archimedean copulas. For extreme-value copulas, the “Pickands dependence function” plays the role of a generator function.

## Usage

```
psi(copula, s)
iPsi(copula, u, ...)
diPsi(copula, u, degree=1, log=FALSE, ...)

A(copula, w)
dAdu(copula, w)
```

## Arguments

|         |                                                                                                                                                     |
|---------|-----------------------------------------------------------------------------------------------------------------------------------------------------|
| copula  | an object of class “copula”.                                                                                                                        |
| u, s, w | numerical vector at which these functions are to be evaluated.                                                                                      |
| ...     | further arguments for specific families.                                                                                                            |
| degree  | the degree of the derivative (defaults to 1).                                                                                                       |
| log     | logical indicating if the <code>log</code> of the <i>absolute</i> derivative is desired. Note that the derivatives of <i>psi</i> alternate in sign. |

## Details

`psi()` and `iPsi()` are, respectively, the generator function  $\psi()$  and its inverse  $\psi^{(-1)}$  for an Archimedean copula, see [pnacopula](#) for definition and more details.

`diPsi()` computes (currently only the first two) derivatives of `iPsi()` ( $= \psi^{(-1)}$ ).

$A()$ , the “Pickands dependence function”, can be seen as the generator function of an extreme-value copula. For instance, in the bivariate case, we have the following result (see, e.g., Gudendorf and Segers 2009):

A bivariate copula  $C$  is an extreme-value copula if and only if

$$C(u, v) = (uv)^{A(\log(v)/\log(uv))}, \quad (u, v) \in (0, 1]^2 \setminus \{(1, 1)\},$$

where  $A : [0, 1] \rightarrow [1/2, 1]$  is convex and satisfies  $\max(t, 1 - t) \leq A(t) \leq 1$  for all  $t \in [0, 1]$ .

In the  $d$ -variate case, there is a similar characterization, except that this time, the Pickands dependence function  $A$  is defined on the  $d$ -dimensional unit simplex.

`dAdu()` returns a data.frame containing the 1st and 2nd derivative of  $A()$ .

## References

Gudendorf, G. and Segers, J. (2010). Extreme-value copulas. In *Copula theory and its applications*, Jaworski, P., Durante, F., Härdle, W. and Rychlik, W., Eds. Springer-Verlag, Lecture Notes in Statistics, 127–146, doi:10.1007/9783642124655; preprint at <https://arxiv.org/abs/0911.1015>.

## See Also

Nonparametric estimators for  $A()$  are available, see [An](#).

## Examples

```
## List the available methods (and their definitions):
showMethods("A")
showMethods("iPsi", incl=TRUE)
```

---

getAcop

*Get "acopula" Family Object by Name*

---

## Description

Get one of our "acopula" family objects (see [acopula-families](#) by name).

Named strings for “translation” between different names and forms of Archimedean copulas.

## Usage

```
getAcop (family, check = TRUE)
getAname(family, objName = FALSE)
```

```
.ac.shortNames
.ac.longNames
.ac.objNames
.ac.classNames
```

**Arguments**

|         |                                                                                                                                                                                                                                                                                                                  |
|---------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| family  | either a <a href="#">character</a> string, the short or longer form of the Archimedean family name (for example, "Clayton" or simply "C"; see the <a href="#">acopula-families</a> documentation), or an <a href="#">acopula</a> family object, or an object inheriting from class <a href="#">archmCopula</a> . |
| check   | logical indicating whether the class of the return value should be checked to be " <a href="#">acopula</a> ".                                                                                                                                                                                                    |
| objName | logical indicating that the <i>name</i> of the R object should be returned, instead of the family name, e.g., "copClayton" instead of "Clayton".                                                                                                                                                                 |

**Value**

getAcop() returns an "[acopula](#)" family object, typically one of one of our predefined ones.

getAname() returns a [character](#) string, the name of an "[acopula](#)" family object.

.as.longnames etc are named string constants, useful in programming for all our (five) standard Archimedean families.

**See Also**

Our predefined [acopula-families](#); the class definition "[acopula](#)".

**Examples**

```
getAcop("Gumbel")

## different ways of getting the same "acopula" family object:
stopifnot(## Joe (three ways):
  identical(getAcop("J"), getAcop("Joe")),
  identical(getAcop("J"), copJoe),
  ## Frank (yet another two different ways):
  identical(getAcop(frankCopula()), copFrank),
  identical(getAcop("frankCopula"), copFrank))

stopifnot(
  identical(getAname(claytonCopula()), getAname("C")),
  identical(getAname(copClayton), "Clayton"), identical(getAname("J"), "Joe"),
  identical(getAname(amhCopula(), TRUE), "copAMH"),
  identical(getAname(joeCopula(), TRUE), "copJoe")
)

.ac.shortNames
.ac.longNames
.ac.objNames
.ac.classNames
```

---

getIniParam

*Get Initial Parameter Estimate for Copula*


---

### Description

A (S4) generic function and methods providing a typically cheap method to get valid parameters for the copula, given the data. This is used, e.g., in `fitCopula()` when `start` is not specified.

### Usage

```
getIniParam(copula, data, default, named = TRUE, ...)
```

### Arguments

|                      |                                                                                                             |
|----------------------|-------------------------------------------------------------------------------------------------------------|
| <code>copula</code>  | a "copula" object.                                                                                          |
| <code>data</code>    | an $n \times d$ -matrix of data to which the copula should be fitted.                                       |
| <code>default</code> | a parameter vector of correct length, to be used when no method is available or the method does "not work". |
| <code>named</code>   | <b>logical</b> indicating if the result should have <b>names</b> .                                          |
| <code>...</code>     | optional further arguments to underlying methods.                                                           |

### Value

a **numeric** vector of correct length, say `param`, which should e.g., "work" in `loglikCopula(param, u = data, copula)`.

### Methods

`signature(copula = "parCopula")` Close to a *default* method (as class "parCopula" contains most copulas), currently mainly trying to use a version of `fitCopula(*, method = "itau")` (itself based on moment matching `iTau()`).

`signature(copula = "mixCopula")` a relatively simple method, for the copula parameters, trying `getInitParam(cop[[k]])` for each component, and using equal weights `w[k]`.

### See Also

`getTheta()` gets such a vector *from* a copula object; `fitCopula`, `loglikCopula`.

### Examples

```
# TODO !
```



---

|          |                                         |
|----------|-----------------------------------------|
| getTheta | <i>Get the Parameter(s) of a Copula</i> |
|----------|-----------------------------------------|

---

**Description**

Get the parameter (vector)  $\theta$  (theta) of a copula, see [setTheta](#) for more background.

**Usage**

```
getTheta(copula, freeOnly = TRUE, attr = FALSE, named = attr)
```

**Arguments**

|          |                                                                                                                                   |
|----------|-----------------------------------------------------------------------------------------------------------------------------------|
| copula   | an R object of class " <a href="#">parCopula</a> ", e.g., " <a href="#">copula</a> ".                                             |
| freeOnly | logical indicating that only non-fixed aka "free" parameters are to be returned (as vector).                                      |
| attr     | logical indicating if <a href="#">attributes</a> (such as lower and upper bounds for each parameters) are to be returned as well. |
| named    | logical if the resulting parameter vector should have <a href="#">names</a> .                                                     |

**Value**

parameter vector of the copula, a [numeric](#) vector, possibly with names and other attributes (depending on the attr and named arguments).

**Methods**

```
signature(copula = "parCopula") a default method, returning numeric(0).
signature(copula = "copula") ..
signature(copula = "acopula") ..
signature(copula = "khoudrajiCopula") ..
signature(copula = "mixCopula") ..
signature(copula = "rotCopula") ..
signature(copula = "Xcopula") ..
```

**See Also**

[setTheta](#), its inverse.

**Examples**

```
showMethods("getTheta")

getTheta(setTheta(copClayton, 0.5)) # is 0.5
```

## Description

Tools for computing a graphical goodness-of-fit (GOF) test based on pairwise Rosenblatt transformed data.

`pairwiseCcop()` computes a  $(n, d, d)$ -array which contains pairwise Rosenblatt-transformed data.

`pairwiseIndepTest()` takes such an array as input and computes a  $(d, d)$ -matrix of test results from pairwise tests of independence (as by `indepTest()`).

`pviTest()` can be used to extract the matrix of p-values from the return matrix of `pairwiseIndepTest()`.

`gpviTest()` takes such a matrix of p-values and computes a global p-value with the method provided.

## Usage

```
pairwiseCcop(u, copula, ...)
pairwiseIndepTest(cu.u, N=256,
  iTTest = indepTestSim(n, p=2, m=2, N=N, verbose = idT.verbose, ...),
  verbose=TRUE, idT.verbose = verbose, ...)

pviTest(piTest)
gpviTest(pvalues, method=p.adjust.methods, globalFun=min)
```

## Arguments

|                          |                                                                                                                                                                                                                                                                                                                             |
|--------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>u</code>           | $(n, d)$ -matrix of copula data.                                                                                                                                                                                                                                                                                            |
| <code>copula</code>      | copula object used for the Rosenblatt transform ( $H_0$ copula).                                                                                                                                                                                                                                                            |
| <code>...</code>         | additional arguments passed to the internal function which computes the conditional copulas (for <code>pairwiseCcop()</code> ). Can be used to pass, for example, the degrees of freedom parameter <code>df</code> for t-copulas.<br>For <code>pairwiseIndepTest()</code> , ... are passed to <code>indepTestSim()</code> . |
| <code>cu.u</code>        | $(n, d, d)$ -array as returned by <code>pairwiseCcop()</code> .                                                                                                                                                                                                                                                             |
| <code>N</code>           | argument of <code>indepTestSim()</code> .                                                                                                                                                                                                                                                                                   |
| <code>iTTest</code>      | the result of (a version of) <code>indepTestSim()</code> ; as it does <i>not</i> depend on the data, and is costly to compute, it can be computed separately and passed here.                                                                                                                                               |
| <code>verbose</code>     | <b>integer</b> (or <b>logical</b> ) indicating if and how much progress should be printed during the computation of the tests for independence.                                                                                                                                                                             |
| <code>idT.verbose</code> | logical, passed as <code>verbose</code> argument to <code>indepTestSim()</code> .                                                                                                                                                                                                                                           |
| <code>piTest</code>      | $(d, d)$ -matrix of <code>indepTest</code> objects as returned by <code>pairwiseIndepTest()</code> .                                                                                                                                                                                                                        |
| <code>pvalues</code>     | $(d, d)$ -matrix of p-values.                                                                                                                                                                                                                                                                                               |

|           |                                                                                                                             |
|-----------|-----------------------------------------------------------------------------------------------------------------------------|
| method    | <a href="#">character</a> vector of adjustment methods for p-values; see <a href="#">p.adjust.methods</a> for more details. |
| globalFun | <a href="#">function</a> determining how to compute a global p-value from a matrix of pairwise adjusted p-values.           |

**Value**

**pairwiseCcop** ( $n, d, d$ )-array `cu.u` with `cu.u[i, j]` containing  $C(u_i | u_j)$  for  $i \neq j$  and  $u_i$  for  $i = j$ .

**pairwiseIndepTest** ( $d, d$ )-matrix of lists with test results as returned by [indepTest\(\)](#). The test results correspond to pairwise tests of independence as conducted by [indepTest\(\)](#).

**pviTest** ( $d, d$ )-matrix of p-values.

**gpviTest** global p-values for the specified methods.

**Note**

If `u` are distributed according to or “perfectly” sampled from a copula, p-values on GOF tests for that copula should be uniformly distributed in  $[0, 1]$ .

**References**

Hofert and Mächler (2014), see [pairsRosenblatt](#).

**See Also**

[pairsRosenblatt](#) for where these tools are used, including [demo\(gof\\_graph\)](#) for examples.

**Examples**

```
## demo(gof_graph)
```

---

 gnacopula

*Goodness-of-fit Testing for (Nested) Archimedean Copulas*


---

**Description**

`gnacopula()` conducts a goodness-of-fit test for the given ( $H_0$ -)copula `cop` based on the (copula-)data `u`.

NOTE: `gnacopula()` is deprecated, call `gofCopula()` instead.

**Usage**

```
gnacopula(u, cop, n.bootstrap,
  estim.method = eval(formals(enacopula)$method),
  include.K=TRUE, n.MC=0, trafo=c("Hering.Hofert", "Rosenblatt"),
  method=eval(formals(gofTstat)$method), verbose=TRUE, ...)
```

**Arguments**

|                           |                                                                                                                                                                                                                                                                                                                                                                                                                      |
|---------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>u</code>            | $n \times d$ -matrix of values in $[0, 1]$ ; should be (pseudo-/copula-)observations from the copula to be tested. Consider applying the function <code>pobs()</code> first in order to obtain <code>u</code> .                                                                                                                                                                                                      |
| <code>cop</code>          | $H_0$ - <code>"outer_nacopula"</code> with specified parameters to be tested for (currently only Archimedean copulas are provided).                                                                                                                                                                                                                                                                                  |
| <code>n.bootstrap</code>  | positive integer specifying the number of bootstrap replicates.                                                                                                                                                                                                                                                                                                                                                      |
| <code>estim.method</code> | <code>character</code> string determining the estimation method; see <code>enacopula()</code> . We currently only recommend the default <code>"mle"</code> (or maybe <code>"smle"</code> ).                                                                                                                                                                                                                          |
| <code>include.K</code>    | logical indicating whether the last component, involving the Kendall distribution function <code>K()</code> , is used in the transformation <code>htrafo()</code> of Hering and Hofert (2011). Note that this only applies to <code>trafo="Hering.Hofert"</code> .                                                                                                                                                   |
| <code>n.MC</code>         | parameter <code>n.MC</code> for <code>htrafo()</code> (and thus for <code>K()</code> ) if <code>trafo="Hering.Hofert"</code> and for <code>cCopula()</code> if <code>trafo="Rosenblatt"</code> .                                                                                                                                                                                                                     |
| <code>trafo</code>        | a <code>character</code> string specifying the multivariate transformation performed for goodness-of-fit testing, which has to be one (or a unique abbreviation) of<br><code>"Hering.Hofert"</code> for the multivariate transformation of Hering and Hofert (2011); see <code>htrafo()</code> .<br><code>"Rosenblatt"</code> for the multivariate transformation of Rosenblatt (1952); see <code>cCopula()</code> . |
| <code>method</code>       | a <code>character</code> string specifying the goodness-of-fit test statistic to be used; see <code>gofTstat()</code> .                                                                                                                                                                                                                                                                                              |
| <code>verbose</code>      | if <code>TRUE</code> , the progress of the bootstrap is displayed via <code>txtProgressBar</code> .                                                                                                                                                                                                                                                                                                                  |
| <code>...</code>          | additional arguments passed to <code>enacopula()</code> .                                                                                                                                                                                                                                                                                                                                                            |

**Details**

The function `gnacopula()` performs a parametric bootstrap for the goodness-of-fit test specified by `trafo` and `method`. The transformation given by `trafo` specifies the multivariate transformation which is first applied to the (copula-) data `u` (typically, the pseudo-observations are used); see `htrafo()` or `cCopula()` for more details. The argument `method` specifies the particular goodness-of-fit test carried out, which is either the Anderson-Darling test for the univariate standard uniform distribution (for `method="AnChisq"` or `method="AnGamma"`) in a one-dimensional setup or the tests described in Genest et al. (2009) for the multivariate standard uniform distribution directly in a multivariate setup. As estimation method, the method provided by `estim.method` is used.

Note that a finite-sample correction is made when computing p-values; see `gofCopula()` for details.

A word of warning: Do work carefully with the variety of different goodness-of-fit tests that can be performed with `gnacopula()`. For example, among the possible estimation methods at hand, only MLE is known to be consistent (under conditions to be verified). Furthermore, for the tests based on the Anderson-Darling test statistic, it is theoretically not clear whether the parametric bootstrap converges. Consequently, the results obtained should be treated with care. Moreover, several estimation methods are known to be prone to numerical errors (see Hofert et al. (2013)) and are thus not recommended to be used in the parametric bootstrap. A warning is given if `gnacopula()` is called with a method not being MLE.

**Value**

gnacopula returns an R object of class "htest". This object contains a list with the bootstrap results including the components

p.value: the bootstrapped p-value;

statistic: the value of the test statistic computed for the data u;

data.name: the name of u;

method: a [character](#) describing the goodness-of-fit test applied;

estimator: the estimator computed for the data u;

bootStats: a list with component estimator containing the estimators for all bootstrap replications and component statistic containing the values of the test statistic for each bootstrap replication.

**References**

Genest, C., Rémillard, B., and Beaudoin, D. (2009), Goodness-of-fit tests for copulas: A review and a power study *Insurance: Mathematics and Economics* **44**, 199–213.

Rosenblatt, M. (1952), Remarks on a Multivariate Transformation, *The Annals of Mathematical Statistics* **23**, 3, 470–472.

Hering, C. and Hofert, M. (2011), Goodness-of-fit tests for Archimedean copulas in large dimensions, submitted.

Hofert, M., Mächler, M., and McNeil, A. J. (2012). Likelihood inference for Archimedean copulas in high dimensions under known margins. *Journal of Multivariate Analysis* **110**, 133–150.

**See Also**

[gofTstat\(\)](#) for the implemented test statistic, [htrafo\(\)](#) and [cCopula\(\)](#) involved and [K\(\)](#) for the Kendall distribution function.

[gofCopula\(\)](#) for other (parametric bootstrap) based goodness-of-fit tests.

---

gofCopula

*Goodness-of-fit Tests for Copulas*


---

**Description**

The goodness-of-fit tests are based, by default, on the empirical process comparing the empirical copula with a parametric estimate of the copula derived under the null hypothesis, the default test statistic, "Sn", being the Cramer-von Mises functional  $S_n$  defined in Equation (2) of Genest, Rémillard and Beaudoin (2009). In that case, approximate p-values for the test statistic can be obtained either using a *parametric bootstrap* (see references two and three) or by means of a faster *multiplier* approach (see references four and five).

Alternative test statistics can be used, in particular if a *parametric bootstrap* is employed.

The principal function is [gofCopula\(\)](#) which, depending on simulation either calls [gofPB\(\)](#) or [gofMB\(\)](#).

**Usage**

```

## Generic [and "rotCopula" method] ----- Main function -----
gofCopula(copula, x, ...)
## S4 method for signature 'copula'
gofCopula(copula, x, N = 1000,
          method = c("Sn", "SnB", "SnC", "Rn"),
          estim.method = c("mpl", "ml", "itau", "irho", "itau.mpl"),
          simulation = c("pb", "mult"), test.method = c("family", "single"),
          verbose = interactive(), ties = NA,
          ties.method = c("max", "average", "first", "last", "random", "min"),
          fit.ties.meth = eval(formals(rank)$ties.method), ...)

## (Deprecated) internal 'helper' functions : ---
gofPB(copula, x, N, method = c("Sn", "SnB", "SnC"),
      estim.method = c("mpl", "ml", "itau", "irho", "itau.mpl"),
      trafo.method = if(method == "Sn") "none" else c("cCopula", "htrafo"),
      trafoArgs = list(), test.method = c("family", "single"),
      verbose = interactive(), useR = FALSE, ties = NA,
      ties.method = c("max", "average", "first", "last", "random", "min"),
      fit.ties.meth = eval(formals(rank)$ties.method), ...)

gofMB(copula, x, N, method = c("Sn", "Rn"),
      estim.method = c("mpl", "ml", "itau", "irho"),
      test.method = c("family", "single"), verbose = interactive(),
      useR = FALSE, m = 1/2, zeta.m = 0, b = 1/sqrt(nrow(x)),
      ties.method = c("max", "average", "first", "last", "random", "min"),
      fit.ties.meth = eval(formals(rank)$ties.method), ...)

```

**Arguments**

|              |                                                                                                                                                                                                                                                                                                   |
|--------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| copula       | object of class " <a href="#">copula</a> " representing the hypothesized copula family.                                                                                                                                                                                                           |
| x            | a data matrix that will be transformed to pseudo-observations using <a href="#">pobs()</a> .                                                                                                                                                                                                      |
| N            | number of bootstrap or multiplier replications to be used to obtain approximate realizations of the test statistic under the null hypothesis.                                                                                                                                                     |
| method       | a <a href="#">character</a> string specifying the goodness-of-fit test statistic to be used. For simulation = "pb", one of "Sn", "SnB" or "SnC" with trafo.method != "none" if method != "Sn". For simulation = "mult", one of "Sn" or "Rn", where the latter is $R_n$ from Genest et al. (2013). |
| estim.method | a <a href="#">character</a> string specifying the estimation method to be used to estimate the dependence parameter(s); see <a href="#">fitCopula()</a> .                                                                                                                                         |
| simulation   | a string specifying the resampling method for generating approximate realizations of the test statistic under the null hypothesis; can be either "pb" (parametric bootstrap) or "mult" (multiplier).                                                                                              |
| test.method  | a <a href="#">character</a> string specifying the test method to be used. Only in exceptional cases should this be different from the default test.method = "family". If                                                                                                                          |

|               |                                                                                                                                                                                                                                                                                                                                                                                                                                     |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|               | test.method = "single", a test precisely for the provided copula (not its parametric family) is conducted. This makes sense only for specific applications such as testing random number generators.                                                                                                                                                                                                                                |
| verbose       | a logical specifying if progress of the parametric bootstrap should be displayed via <a href="#">txtProgressBar</a> .                                                                                                                                                                                                                                                                                                               |
| ...           | for <code>gofCopula</code> , additional arguments passed to <code>gofPB()</code> or <code>gofMB()</code> ; for <code>gofPB()</code> and <code>gofMB()</code> : additional arguments passed to <code>fitCopula()</code> . These may notably contain <code>hideWarnings</code> , and <code>optim.method</code> , <code>optim.control</code> , <code>lower</code> , or <code>upper</code> depending on the <code>optim.method</code> . |
| trafo.method  | only for the parametric bootstrap ("pb"): String specifying the transformation to $U[0, 1]^d$ ; either "none" or one of "cCopula", see <code>cCopula()</code> , or "htrafo", see <code>htrafo()</code> . If <code>method != "Sn"</code> , one needs to set <code>trafo.method != "none"</code> .                                                                                                                                    |
| trafoArgs     | only for the parametric bootstrap. A <a href="#">list</a> of optional arguments passed to the transformation method (see <code>trafo.method</code> above).                                                                                                                                                                                                                                                                          |
| useR          | logical indicating whether an R or C implementation is used.                                                                                                                                                                                                                                                                                                                                                                        |
| ties.method   | string specifying how ranks should be computed, except for fitting, if there are ties in any of the coordinate samples of <code>x</code> ; passed to <code>pobs</code> .                                                                                                                                                                                                                                                            |
| fit.ties.meth | string specifying how ranks should be computed when fitting by maximum pseudo-likelihood if there are ties in any of the coordinate samples of <code>x</code> ; passed to <code>pobs</code> .                                                                                                                                                                                                                                       |
| ties          | only for the parametric bootstrap. Logical indicating whether a version of the parametric bootstrap adapted to the presence of ties in any of the coordinate samples of <code>x</code> should be used; the default value of NA indicates that the presence/absence of ties will be checked for automatically.                                                                                                                       |
| m, zeta.m     | only for the multiplier with <code>method = "Rn"</code> . <code>m</code> is the power and <code>zeta.m</code> is the adjustment parameter $\zeta_m$ for the denominator of the test statistic.                                                                                                                                                                                                                                      |
| b             | only for the multiplier. <code>b</code> is the bandwidth required for the estimation of the first-order partial derivatives based on the empirical copula.                                                                                                                                                                                                                                                                          |

## Details

If the parametric bootstrap is used, the dependence parameters of the hypothesized copula family can be estimated by any estimation method available for the family, up to a few exceptions. If the multiplier is used, any of the rank-based methods can be used in the bivariate case, but only maximum pseudo-likelihood estimation can be used in the multivariate (multiparameter) case.

The price to pay for the higher computational efficiency of the multiplier is more programming work as certain partial derivatives need to be computed for each hypothesized parametric copula family. When estimation is based on maximization of the pseudo-likelihood, these have been implemented for six copula families thus far: the Clayton, Gumbel-Hougaard, Frank, Plackett, normal and  $t$  copula families. For other families, numerical differentiation based on `grad()` from package **numDeriv** is used (and a warning message is displayed).

Although the empirical processes involved in the multiplier and the parametric bootstrap-based test are asymptotically equivalent under the null, the finite-sample behavior of the two tests might differ significantly.

Both for the parametric bootstrap and the multiplier, the approximate p-value is computed as

$$(0.5 + \sum_{b=1}^N \mathbf{1}_{\{T_b \geq T\}}) / (N + 1),$$

where  $T$  and  $T_b$  denote the test statistic and the bootstrapped test statistic, respectively. This ensures that the approximate p-value is a number strictly between 0 and 1, which is sometimes necessary for further treatments. See Pesarin (2001) for more details.

For the normal and  $t$  copulas, several dependence structures can be hypothesized: "ex" for exchangeable, "ar1" for AR(1), "toep" for Toeplitz, and "un" for unstructured (see `ellipCopula()`). For the  $t$  copula, "df.fixed" has to be set to TRUE, which implies that the degrees of freedom are not considered as a parameter to be estimated.

The former argument `print.every` is deprecated and not supported anymore; use `verbose` instead.

### Value

An object of class `htest` which is a list, some of the components of which are

|                        |                                                                 |
|------------------------|-----------------------------------------------------------------|
| <code>statistic</code> | value of the test statistic.                                    |
| <code>p.value</code>   | corresponding approximate p-value.                              |
| <code>parameter</code> | estimates of the parameters for the hypothesized copula family. |

### Note

These tests were theoretically studied and implemented under the assumption of continuous margins, which implies that ties in the component samples occur with probability zero. The presence of ties in the data might substantially affect the approximate p-values. Through argument `ties`, the user can however select a version of the parametric bootstrap adapted to the presence of ties. No such adaption exists for the multiplier for the moment.

### References

- Genest, C., Huang, W., and Dufour, J.-M. (2013). A regularized goodness-of-fit test for copulas. *Journal de la Société française de statistique* **154**, 64–77.
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- Genest, C., Rémillard, B., and Beaudoin, D. (2009). Goodness-of-fit tests for copulas: A review and a power study. *Insurance: Mathematics and Economics* **44**, 199–214.
- Kojadinovic, I., Yan, J., and Holmes M. (2011). Fast large-sample goodness-of-fit tests for copulas. *Statistica Sinica* **21**, 841–871.
- Kojadinovic, I. and Yan, J. (2011). A goodness-of-fit test for multivariate multiparameter copulas based on multiplier central limit theorems. *Statistics and Computing* **21**, 17–30.
- Kojadinovic, I. and Yan, J. (2010). Modeling Multivariate Distributions with Continuous Margins Using the copula R Package. *Journal of Statistical Software* **34**(9), 1–20, <https://www.jstatsoft.org/v34/i09/>.



Kojadinovic, I. (2017). Some copula inference procedures adapted to the presence of ties. *Computational Statistics and Data Analysis* **112**, 24–41, <https://arxiv.org/abs/1609.05519>.

Pesarin, F. (2001). *Multivariate Permutation Tests: With Applications in Biostatistics*. Wiley.

### See Also

`fitCopula()` for the underlying estimation procedure and `gofTstat()` for details on \*some\* of the available test statistics.

### Examples

```
## The following example is available in batch through
## demo(gofCopula)

n <- 200; N <- 1000 # realistic (but too large for interactive use)
n <- 60; N <- 200 # (time (and tree !) saving ...)

## A two-dimensional data example -----
set.seed(271)
x <- rCopula(n, claytonCopula(3))

## Does the Gumbel family seem to be a good choice (statistic "Sn")?
gofCopula(gumbelCopula(), x, N=N)
## With "SnC", really s..l..o..w.. --- with "SnB", *EVEN* slower
gofCopula(gumbelCopula(), x, N=N, method = "SnC", trafo.method = "cCopula")
## What about the Clayton family?
gofCopula(claytonCopula(), x, N=N)

## Similar with a different estimation method
gofCopula(gumbelCopula(), x, N=N, estim.method="itau")
gofCopula(claytonCopula(), x, N=N, estim.method="itau")

## A three-dimensional example -----
x <- rCopula(n, tCopula(c(0.5, 0.6, 0.7), dim = 3, dispstr = "un"))

## Does the Gumbel family seem to be a good choice?
g.copula <- gumbelCopula(dim = 3)
gofCopula(g.copula, x, N=N)
## What about the t copula?
t.copula <- tCopula(dim = 3, dispstr = "un", df.fixed = TRUE)
if(FALSE) ## this is *VERY* slow currently
  gofCopula(t.copula, x, N=N)

## The same with a different estimation method
gofCopula(g.copula, x, N=N, estim.method="itau")
if(FALSE) # still really slow
  gofCopula(t.copula, x, N=N, estim.method="itau")

## The same using the multiplier approach
gofCopula(g.copula, x, N=N, simulation="mult")
gofCopula(t.copula, x, N=N, simulation="mult")
```

```
if(FALSE) # no yet possible
  gofCopula(t.copula, x, N=N, simulation="mult", estim.method="itau")
```

gofEVCopula

*Goodness-of-fit Tests for Bivariate Extreme-Value Copulas***Description**

Goodness-of-fit tests for extreme-value copulas based on the empirical process comparing one of the two nonparametric rank-based estimator of the Pickands dependence function studied in Genest and Segers (2009) with a parametric estimate of the Pickands dependence function derived under the null hypothesis. The test statistic is the Cramer-von Mises functional  $S_n$  defined in Equation (5) of Genest, Kojadinovic, G. Nešlehová, and Yan (2010). Approximate p-values for the test statistic are obtained using a parametric bootstrap.

**Usage**

```
gofEVCopula(copula, x, N = 1000,
            method = c("mpl", "ml", "itau", "irho"),
            estimator = c("CFG", "Pickands"), m = 1000,
            verbose = interactive(),
            ties.method = c("max", "average", "first", "last", "random", "min"),
            fit.ties.meth = eval(formals(rank)$ties.method), ...)
```

**Arguments**

|               |                                                                                                                                                                                                     |
|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| copula        | object of class " <a href="#">evCopula</a> " representing the hypothesized extreme-value copula family.                                                                                             |
| x             | a data matrix that will be transformed to pseudo-observations.                                                                                                                                      |
| N             | number of bootstrap samples to be used to simulate realizations of the test statistic under the null hypothesis.                                                                                    |
| method        | estimation method to be used to estimate the dependence parameter(s); can be either "mpl" (maximum pseudo-likelihood), "itau" (inversion of Kendall's tau) or "irho" (inversion of Spearman's rho). |
| estimator     | specifies which nonparametric rank-based estimator of the unknown Pickands dependence function to use; can be either "CFG" (Caperaa-Fougeres-Genest) or "Pickands".                                 |
| m             | number of points of the uniform grid on [0,1] used to compute the test statistic numerically.                                                                                                       |
| verbose       | a logical specifying if progress of the bootstrap should be displayed via <a href="#">txtProgressBar</a> .                                                                                          |
| ties.method   | string specifying how ranks should be computed, except for fitting, if there are ties in any of the coordinate samples of x; passed to <a href="#">pobs</a> .                                       |
| fit.ties.meth | string specifying how ranks should be computed when fitting by maximum pseudo-likelihood if there are ties in any of the coordinate samples of x; passed to <a href="#">pobs</a> .                  |

... further optional arguments, passed to `fitCopula()`, notably `optim.method`, the method for `optim()`. In **copula** versions 0.999-14 and earlier, the default for that was "Nelder-Mead", but now is the same as for `fitCopula()`.

### Details

More details can be found in the second reference.

The former argument `print.every` is deprecated and not supported anymore; use `verbose` instead.

### Value

An object of `class` `hctest` which is a list, some of the components of which are

|                        |                                                                 |
|------------------------|-----------------------------------------------------------------|
| <code>statistic</code> | value of the test statistic.                                    |
| <code>p.value</code>   | corresponding approximate p-value.                              |
| <code>parameter</code> | estimates of the parameters for the hypothesized copula family. |

### Note

For a given degree of dependence, the most popular bivariate extreme-value copulas are strikingly similar.

### References

Genest, C. and Segers, J. (2009). Rank-based inference for bivariate extreme-value copulas. *Annals of Statistics* **37**, 2990–3022.

Genest, C. Kojadinovic, I., G. Nešlehová, J., and Yan, J. (2011). A goodness-of-fit test for bivariate extreme-value copulas. *Bernoulli* **17**(1), 253–275.

### See Also

[evCopula](#), [evTestC](#), [evTestA](#), [evTestK](#), [gofCopula](#), [An](#).

### Examples

```
n <- 100; N <- 1000 # realistic (but too large currently for CRAN checks)
n <- 60; N <- 200 # (time (and tree !) saving ...)
x <- rCopula(n, claytonCopula(3))

## Does the Gumbel family seem to be a good choice?
gofEVCopula(gumbelCopula(), x, N=N)

## The same with different (and cheaper) estimation methods:
gofEVCopula(gumbelCopula(), x, N=N, method="itau")
gofEVCopula(gumbelCopula(), x, N=N, method="irho")

## The same with different extreme-value copulas
```

```

gofEVCopula(galambosCopula(), x, N=N)
gofEVCopula(galambosCopula(), x, N=N, method="itau")
gofEVCopula(galambosCopula(), x, N=N, method="irho")

gofEVCopula(huslerReissCopula(), x, N=N)
gofEVCopula(huslerReissCopula(), x, N=N, method="itau")
gofEVCopula(huslerReissCopula(), x, N=N, method="irho")

gofEVCopula(tevCopula(df.fixed=TRUE), x, N=N)
gofEVCopula(tevCopula(df.fixed=TRUE), x, N=N, method="itau")
gofEVCopula(tevCopula(df.fixed=TRUE), x, N=N, method="irho")

```

---

gofOtherTstat

*Various Goodness-of-fit Test Statistics*


---

### Description

gofBTstat() computes supposedly Beta distributed test statistics for checking uniformity of  $u$  on the unit sphere.

### Usage

```
gofBTstat(u)
```

### Arguments

$u$   $(n, d)$ -matrix of values whose rows supposedly follow a uniform distribution on the unit sphere in  $\mathbf{R}^d$ .

### Value

An  $(n, d - 1)$ -matrix where the  $(i, k)$ th entry is

$$B_{ik} = \frac{\sum_{j=1}^k u_{ij}^2}{\sum_{j=1}^d u_{ij}^2}.$$

### References

Li, R.-Z., Fang, K.-T., and Zhu, L.-X. (1997). Some Q-Q probability plots to test spherical and elliptical symmetry. *Journal of Computational and Graphical Statistics* **6**(4), 435–450.

### Examples

```

## generate data on the unit sphere
n <- 360
d <- 5
set.seed(1)
x <- matrix(rnorm(n*d), ncol=d)

```

```

U <- x/sqrt(rowSums(x^2))

## compute the test statistics B_k, k in {1,..,d-1}
Bmat <- gofBTstat(U)

## (graphically) check if Bmat[,k] follows a Beta(k/2, (d-k)/2) distribution
qqp <- function(k, Bmat) {
  d <- ncol(Bmat)+1L
  tit <- substitute(plain("Beta")(A.,.B.)~~ bold("Q-Q Plot"),
    list(.A. = k/2, .B. = (d-k)/2))
  qqplot2(Bmat[,k], qf=function(p) qbeta(p, shape1=k/2, shape2=(d-k)/2),
    main.args=list(text=tit, side=3, cex=1.3, line=1.1, xpd=NA))
}
qqp(1, Bmat=Bmat) # k=1
qqp(3, Bmat=Bmat) # k=3

```

gofTstat

*Goodness-of-fit Test Statistics***Description**

gofTstat() computes various goodness-of-fit test statistics typically used in [gofCopula\\*](#), simulation = "pb"). gofT2stat() computes the two-sample goodness of fit test statistic of Rémillard and Scaillet (2009).

**Usage**

```

gofTstat(u, method = c("Sn", "SnB", "SnC", "AnChisq", "AnGamma"),
  useR = FALSE, ...)
gofT2stat(u1, u2, useR = FALSE)

```

**Arguments**

|        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|--------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| u      | $n \times d$ -matrix of values in $[0, 1]$ , supposedly independent uniform observations in the hypercube, that is, $U_i \sim U[0, 1]^d$ , i.i.d., for $i \in \{1, \dots, n\}$ .                                                                                                                                                                                                                                                                                                                          |
| u1, u2 | $n \times d$ -matrices of copula samples to be compare against. The two matrices must have an equal number of columns $d$ but can differ in $n$ (number of rows).                                                                                                                                                                                                                                                                                                                                         |
| method | a <b>character</b> string specifying the goodness-of-fit test statistic to be used, which has to be one (or a unique abbreviation) of <ul style="list-style-type: none"> <li>"Sn" for computing the test statistic <math>S_n</math> from Genest, Rémillard, Beaudoin (2009).</li> <li>"SnB" for computing the test statistic <math>S_n^{(B)}</math> from Genest, Rémillard, Beaudoin (2009).</li> <li>"SnC" for computing the test statistic <math>S_n^{(C)}</math> from Genest et al. (2009).</li> </ul> |

"AnChisq" Anderson-Darling test statistic for computing (supposedly)  $U[0, 1]$ -distributed (under  $H_0$ ) random variates via the distribution function of the chi-square distribution with  $d$  degrees of freedom. To be more precise, the Anderson-Darling test statistic of the variates

$$\chi_d^2 \left( \sum_{j=1}^d (\Phi^{-1}(u_{ij}))^2 \right)$$

is computed (via `ADGofTest::ad.test`), where  $\Phi^{-1}$  denotes the quantile function of the standard normal distribution function,  $\chi_d^2$  denotes the distribution function of the chi-square distribution with  $d$  degrees of freedom, and  $u_{ij}$  is the  $j$ th component in the  $i$ th row of  $u$ .

"AnGamma" similar to `method="AnChisq"` but based on the variates

$$\Gamma_d \left( \sum_{j=1}^d (-\log u_{ij}) \right),$$

where  $\Gamma_d$  denotes the distribution function of the gamma distribution with shape parameter  $d$  and shape parameter one (being equal to an  $\text{Erlang}(d)$  distribution function).

`useR` logical indicating whether an R or C implementation is used.  
`...` additional arguments passed for computing the different test statistics.

### Details

These functions should be used with care. The different test statistics were implemented (partly) for different purposes and goodness-of-fit tests and should be used only with knowledge about such tests (see the references for more details).

### Value

The value of the test statistic, a `numeric`.

### References

- Genest, C., Rémillard, B., and Beaudoin, D. (2009), Goodness-of-fit tests for copulas: A review and a power study *Insurance: Mathematics and Economics* **44**, 199–213.
- Rosenblatt, M. (1952), Remarks on a Multivariate Transformation, *The Annals of Mathematical Statistics* **23**, 3, 470–472.
- Hering, C. and Hofert, M. (2014), Goodness-of-fit tests for Archimedean copulas in high dimensions, *Innovations in Quantitative Risk Management*.
- Hofert, M., Mächler, M., and McNeil, A. J. (2012). Likelihood inference for Archimedean copulas in high dimensions under known margins. *Journal of Multivariate Analysis* **110**, 133–150.
- Rémillard, B. and Scaillet, O. (2009). Testing for equality between two copulas. *Journal of Multivariate Analysis* **100**, 377–386.

**See Also**

[gofCopula\(\)](#) for goodness-of-fit tests where (some of) the test statistics of [gofTstat\(\)](#) are used.

**Examples**

```
## generate data
cop <- archmCopula("Gumbel", param=iTau(gumbelCopula(), 0.5), dim=5)
set.seed(1)
U <- rCopula(1000, cop)

## compute Sn (as is done in a parametric bootstrap, for example)
Uhat <- pobs(U) # pseudo-observations
u <- cCopula(Uhat, copula = cop) # Rosenblatt transformed data (with correct copula)
gofTstat(u, method = "Sn", copula = cop) # compute test statistic Sn; requires copula argument
```

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htrafo

*GOF Testing Transformation of Hering and Hofert*


---

**Description**

The transformation described in Hering and Hofert (2014), for Archimedean copulas.

**Usage**

```
htrafo(u, copula, include.K = TRUE, n.MC = 0, inverse = FALSE,
       method = eval(formals(qK)$method), u.grid, ...)
```

**Arguments**

|           |                                                                                                                                                                                                                                                                                                                                                                                                                                            |
|-----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| u         | $n \times d$ -matrix with values in $[0, 1]$ . If <code>inverse=FALSE</code> (the default), u contains (pseudo-/copula-)observations from the copula <code>copula</code> based on which the transformation is carried out; consider applying the function <a href="#">pobs()</a> first in order to obtain u. If <code>inverse=TRUE</code> , u contains $U[0, 1]^d$ distributed values which are transformed to copula-based (copula) ones. |
| copula    | an Archimedean copula specified as " <a href="#">outer_nacopula</a> " or " <a href="#">archmCopula</a> ".                                                                                                                                                                                                                                                                                                                                  |
| include.K | logical indicating whether the last component, involving the Kendall distribution function <a href="#">K</a> , is used in <a href="#">htrafo()</a> .                                                                                                                                                                                                                                                                                       |
| n.MC      | parameter n.MC for <a href="#">K</a> .                                                                                                                                                                                                                                                                                                                                                                                                     |
| inverse   | logical indicating whether the inverse of the transformation is returned.                                                                                                                                                                                                                                                                                                                                                                  |
| method    | method to compute <a href="#">qK()</a> .                                                                                                                                                                                                                                                                                                                                                                                                   |
| u.grid    | argument of <a href="#">qK()</a> (for <code>method="discrete"</code> ).                                                                                                                                                                                                                                                                                                                                                                    |
| ...       | additional arguments passed to <a href="#">qK()</a> if <code>inverse = TRUE</code> .                                                                                                                                                                                                                                                                                                                                                       |

## Details

Given a  $d$ -dimensional random vector  $\mathbf{U}$  following an Archimedean copula  $C$  with generator  $\psi$ , Hering and Hofert (2014) showed that  $\mathbf{U}' \sim U[0, 1]^d$ , where

$$U'_j = \left( \frac{\sum_{k=1}^j \psi^{-1}(U_k)}{\sum_{k=1}^{j+1} \psi^{-1}(U_k)} \right)^j, \quad j \in \{1, \dots, d-1\}, \quad U'_d = K(C(\mathbf{U})).$$

htrafo applies this transformation row-wise to  $\mathbf{u}$  and thus returns either an  $n \times d$ - or an  $n \times (d-1)$ -matrix, depending on whether the last component  $U'_d$  which involves the (possibly numerically challenging) Kendall distribution function  $K$  is used (`include.K=TRUE`) or not (`include.K=FALSE`).

## Value

htrafo() returns an  $n \times d$ - or  $n \times (d-1)$ -matrix (depending on whether `include.K` is `TRUE` or `FALSE`) containing the transformed input  $\mathbf{u}$ .

## References

Hering, C. and Hofert, M. (2014). Goodness-of-fit tests for Archimedean copulas in high dimensions. *Innovations in Quantitative Risk Management*.

## Examples

```
## Sample and build pseudo-observations (what we normally have available)
## of a Clayton copula
tau <- 0.5
theta <- iTau(claytonCopula(), tau = tau)
d <- 5
cc <- claytonCopula(theta, dim = d)
set.seed(271)
n <- 1000
U <- rCopula(n, copula = cc)
X <- qnorm(U) # X now follows a meta-Gumbel model with N(0,1) marginals
U <- pobs(X) # build pseudo-observations

## Graphically check if the data comes from a meta-Clayton model
## with the transformation of Hering and Hofert (2014):
U.H <- htrafo(U, copula = cc) # transform the data
splom2(U.H, cex = 0.2) # looks good

## The same for an 'outer_nacopula' object
cc. <- onacopulaL("Clayton", list(theta, 1:d))
U.H. <- htrafo(U, copula = cc.)
splom2(U.H., cex = 0.2) # looks good

## What about a meta-Gumbel model?
## The parameter is chosen such that Kendall's tau equals (the same) tau
gc <- gumbelCopula(iTau(gumbelCopula(), tau = tau), dim = d)

## Plot of the transformed data (Hering and Hofert (2014)) to see the
## deviations from uniformity
```



```
U.H.. <- htrafo(U, copula = gc)
splom2(U.H., cex = 0.2) # deviations visible
```

---

|             |                                                    |
|-------------|----------------------------------------------------|
| indepCopula | <i>Construction of Independence Copula Objects</i> |
|-------------|----------------------------------------------------|

---

### Description

Constructs an independence copula with its corresponding dimension.

### Usage

```
indepCopula(dim = 2)
```

### Arguments

dim                    the dimension of the copula.

### Value

An independence copula object of class "[indepCopula](#)".

### See Also

Mathematically, the independence copula is also a special (boundary) case of e.g., classes "[archmCopula](#)", "[ellipCopula](#)", and "[evCopula](#)".

### Examples

```
indep.cop <- indepCopula(3)
x <- rCopula(10, indep.cop)
dCopula(x, indep.cop)
persp(indepCopula(), pCopula)
```

---

|                   |                            |
|-------------------|----------------------------|
| indepCopula-class | <i>Class "indepCopula"</i> |
|-------------------|----------------------------|

---

### Description

Independence copula class.

### Objects from the Class

Objects can be created by calls of the form `new("indepCopula", ...)` or by function [indepCopula\(\)](#). Such objects can be useful as special cases of parametric copulas, bypassing copula-specific computations such as distribution, density, and sampler.

**Slots**

**dimension:** Object of class "numeric", dimension of the copula.

**exprdist:** an [expression](#) of length two, for the "formulas" of the cdf and pdf of the copula.

**Methods**

**A** signature(copula = "indepCopula"): ...

**dCopula** signature(copula = "indepCopula"): ...

**pCopula** signature(copula = "indepCopula"): ...

**rCopula** signature(copula = "indepCopula"): ...

**Extends**

Class "indepCopula" directly extends classes "[dimCopula](#)" and "[parCopula](#)".

**See Also**

[indepCopula](#); documentation for classes [dimCopula](#) and [parCopula](#).

**Examples**

```
getClass("indepCopula")
```

---

indepTest

*Test Independence of Continuous Random Variables via Empirical Copula*

---

**Description**

Multivariate independence test based on the empirical copula process as proposed by Christian Genest and Bruno Rémillard. The test can be seen as composed of three steps: (i) a simulation step, which consists of simulating the distribution of the test statistics under independence for the sample size under consideration; (ii) the test itself, which consists of computing the approximate p-values of the test statistics with respect to the empirical distributions obtained in step (i); and (iii) the display of a graphic, called a *dependogram*, enabling to understand the type of departure from independence, if any. More details can be found in the articles cited in the reference section.

**Usage**

```
indepTestSim(n, p, m = p, N = 1000, verbose = interactive())
indepTest(x, d, alpha=0.05)
dependogram(test, pvalues = FALSE, print = FALSE)
```

**Arguments**

|         |                                                                                                                                                                                       |
|---------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| n       | sample size when simulating the distribution of the test statistics under independence.                                                                                               |
| p       | dimension of the data when simulating the distribution of the test statistics under independence.                                                                                     |
| m       | maximum cardinality of the subsets of variables for which a test statistic is to be computed. It makes sense to consider $m \ll p$ especially when p is large.                        |
| N       | number of repetitions when simulating under independence.                                                                                                                             |
| verbose | a logical specifying if progress should be displayed via <code>txtProgressBar</code> .                                                                                                |
| x       | data frame or data matrix containing realizations (one per line) of the random vector whose independence is to be tested.                                                             |
| d       | object of class "indepTestDist" as returned by the function <code>indepTestSim()</code> . It can be regarded as the empirical distribution of the test statistics under independence. |
| alpha   | significance level used in the computation of the critical values for the test statistics.                                                                                            |
| test    | object of class "indepTest" as returned by <code>indepTest()</code> .                                                                                                                 |
| pvalues | logical indicating whether the dependogram should be drew from test statistics or the corresponding p-values.                                                                         |
| print   | logical indicating whether details should be printed.                                                                                                                                 |

**Details**

The current (C code) implementation of `indepTestSim()` uses (RAM) memory of size  $O(n^2p)$ , and time  $O(Nn^2p)$ . This renders it unfeasible when n is large.

See the references below for more details, especially Genest and Rémillard (2004).

The former argument `print.every` is deprecated and not supported anymore; use `verbose` instead.

**Value**

The function `indepTestSim()` returns an object of class "indepTestDist" whose attributes are: `sample.size`, `data.dimension`, `max.card.subsets`, `number.repetitions`, `subsets` (list of the subsets for which test statistics have been computed), `subsets.binary` (subsets in binary 'integer' notation), `dist.statistics.independence` (a N line matrix containing the values of the test statistics for each subset and each repetition) and `dist.global.statistic.independence` (a vector a length N containing the values of the global Cramér-von Mises test statistic for each repetition – see Genest *et al* (2007), p.175).

The function `indepTest()` returns an object of class "indepTest" whose attributes are: `subsets`, `statistics`, `critical.values`, `pvalues`, `fisher.pvalue` (a p-value resulting from a combination *à la* Fisher of the subset statistic p-values), `tippett.pvalue` (a p-value resulting from a combination *à la* Tippett of the subset statistic p-values), `alpha` (global significance level of the test), `beta` (1 - beta is the significance level per statistic), `global.statistic` (value of the global Cramér-von Mises statistic derived directly from the independence empirical copula process - see Genest *et al* (2007), p.175) and `global.statistic.pvalue` (corresponding p-value).

## References

- Deheuvels, P. (1979). La fonction de dépendance empirique et ses propriétés: un test non paramétrique d'indépendance, *Acad. Roy. Belg. Bull. Cl. Sci.*, 5th Ser. **65**, 274–292.
- Deheuvels, P. (1981) A non parametric test for independence, *Publ. Inst. Statist. Univ. Paris.* **26**, 29–50.
- Genest, C. and Rémillard, B. (2004) Tests of independence and randomness based on the empirical copula process. *Test* **13**, 335–369.
- Genest, C., Quessy, J.-F., and Rémillard, B. (2006). Local efficiency of a Cramer-von Mises test of independence, *Journal of Multivariate Analysis* **97**, 274–294.
- Genest, C., Quessy, J.-F., and Rémillard, B. (2007) Asymptotic local efficiency of Cramér-von Mises tests for multivariate independence. *The Annals of Statistics* **35**, 166–191.

## See Also

[serialIndepTest](#), [multIndepTest](#), [multSerialIndepTest](#).

## Examples

```
## Consider the following example taken from
## Genest and Remillard (2004), p 352:

set.seed(2004)
x <- matrix(rnorm(500),100,5)
x[,1] <- abs(x[,1]) * sign(x[,2] * x[,3])
x[,5] <- x[,4]/2 + sqrt(3) * x[,5]/2

## In order to test for independence "within" x, the first step consists
## in simulating the distribution of the test statistics under
## independence for the same sample size and dimension,
## i.e. n=100 and p=5. As we are going to consider all the subsets of
## {1,...,5} whose cardinality is between 2 and 5, we set p=m=5.

## For a realistic N = 1000 (default), this takes a few seconds:
N. <- if(copula:::doExtras()) 1000 else 120
N.

system.time(d <- indepTestSim(100, 5, N = N.))
## For N=1000, 2 seconds (lynne 2015)
## You could save 'd' for future use, via saveRDS()

## The next step consists of performing the test itself (and print its results):
(iTst <- indepTest(x,d))

## Display the dependogram with the details:
dependogram(iTst, print=TRUE)

## We could have tested for a weaker form of independence, for instance,
## by only computing statistics for subsets whose cardinality is between 2
## and 3. Consider for instance the following data:
y <- matrix(runif(500),100,5)
## and perform the test:
```

```

system.time( d <- indepTestSim(100,5,3, N=N.) )
iTy <- indepTest(y,d)
iTy
dependogram(iTy, print=TRUE)

```

---

|         |                                                                                  |
|---------|----------------------------------------------------------------------------------|
| initOpt | <i>Initial Interval or Value for Parameter Estimation of Archimedean Copulas</i> |
|---------|----------------------------------------------------------------------------------|

---

### Description

Compute an initial interval or initial value for optimization/estimation routines (only a heuristic; if this fails, choose your own interval or value).

### Usage

```

initOpt(family, tau.range=NULL, interval = TRUE, u,
        method = c("tau.Gumbel", "tau.mean"), warn = TRUE, ...)

```

### Arguments

|           |                                                                                                                                                                                                                                                                                                                                                                |
|-----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| family    | Archimedean family to find an initial interval for.                                                                                                                                                                                                                                                                                                            |
| tau.range | numeric vector containing lower and upper admissible Kendall's tau, or NULL which chooses family-specific defaults, see the function definition.                                                                                                                                                                                                               |
| interval  | <b>logical</b> indicating whether an initial interval (the default) or an initial value should be returned.                                                                                                                                                                                                                                                    |
| u         | matrix of realizations following the copula family specified by family. Note that u can be omitted if interval=TRUE.                                                                                                                                                                                                                                           |
| method    | a <b>character</b> string specifying the method to be used to compute an estimate of Kendall's tau. This has to be one (or a unique abbreviation) of<br>"tau.Gumbel" an estimator based on the diagonal maximum-likelihood estimator for Gumbel is used.<br>"tau.mean" an estimator based on the mean of pairwise sample versions of Kendall's tau is applied. |
| warn      | logical indicating if warnings are printed for method="tau.Gumbel" when the diagonal maximum-likelihood estimator is smaller than 1.                                                                                                                                                                                                                           |
| ...       | additional arguments passed to cor() when method="tau.mean". Note that otherwise (no additional arg.), the much faster <b>cor.fk()</b> from package <b>pcaPP</b> is used.                                                                                                                                                                                      |

## Details

For `method="tau.mean"` and `interval=FALSE`, the mean of pairwise sample versions of Kendall's tau is computed as an estimator of the Kendall's tau of the Archimedean copula family provided. This can be slow, especially if the dimension is large. Method `method="tau.Gumbel"` (the default) uses the explicit and thus very fast diagonal maximum-likelihood estimator for Gumbel's family to find initial values. Given this estimator  $\hat{\theta}^G$ , the corresponding Kendall's tau is  $\tau^G(\hat{\theta}^G)$  where  $\tau^G(\theta) = (\theta - 1)/\theta$  denotes Kendall's tau for Gumbel. This provides an estimator of Kendall's tau which is typically much faster to evaluate than, pairwise Kendall's taus. Given the estimated 'amount of concordance' based on Kendall's tau, one can obtain an initial value for the provided family by applying  $\tau^{-1}$ , that is, the inverse of Kendall's tau of the family for which the initial value is to be computed. Note that if the estimated Kendall's tau does not lie in the range of Kendall's tau as provided by the bivariate vector `tau.range`, the point in `tau.range` closest to the estimated Kendall's tau is chosen.

The default (`interval=TRUE`) returns a reasonably large initial interval; see the default of `tau.range` in the definition of `initOpt` for the chosen values (in terms of Kendall's tau). These default values cover a large range of concordance. If this interval is (still) too small, one can adjust it by providing `tau.range`. If it is too large, a 'distance to concordance' can be used to determine parameter values such that the corresponding Kendall's taus share a certain distance to the initial value. For more details, see Hofert et al. (2012). Finally, let us note that for the case `interval=TRUE`, `u` is not required.

## Value

initial interval which can be used for optimization (for example, for [emle](#)).

## References

Hofert, M., Mächler, M., and McNeil, A. J. (2012). Likelihood inference for Archimedean copulas in high dimensions under known margins. *Journal of Multivariate Analysis* **110**, 133–150.

## See Also

[enacopula](#), [emle](#), [edmlc](#), [emde](#), and [ebeta](#) (where `initOpt` is applied to find initial intervals).

## Examples

```
## Definition of the function:
initOpt

## Generate some data:
tau <- 0.25
(theta <- copGumbel@iTau(tau)) # 4/3
d <- 20
(cop <- onacopulaL("Gumbel", list(theta,1:d)))

set.seed(1)
n <- 200
U <- rnacopula(n, cop)

## Initial interval:
```

```
initOpt("Gumbel") # contains theta

## Initial values:
initOpt("Gumbel", interval=FALSE, u=U) # 1.3195
initOpt("Gumbel", interval=FALSE, u=U, method="tau.mean") # 1.2844
```

---

|          |                                           |
|----------|-------------------------------------------|
| interval | <i>Construct Simple "interval" Object</i> |
|----------|-------------------------------------------|

---

## Description

Easy construction of an object of class `interval`, using typical mathematical notation.

## Usage

```
interval(ch)
```

## Arguments

ch                    a character string specifying the interval.

## Value

an `interval` object.

## See Also

the `interval` class documentation, notably its reference to more sophisticated interval classes available for R.

## Examples

```
interval("[0, 1)")

## Two ways to specify open interval borders:
identical(interval("]-1,1["),
           interval("(-1,1)"))

## infinite :
interval("[0, Inf)")

## arithmetic with scalars works:
4 + 2 * interval("[0, 1.5)") # -> [4, 7)

## str() to look at internals:
str( interval("[1.2, 7]") )
```

---

interval-class      *Class "interval" of Simple Intervals*

---

### Description

The S4 `class` "interval" is a simple class for numeric intervals.

"maybeInterval" is a class union (see `setClassUnion`) of "interval" and "NULL".

### Objects from the Class

Objects can be created by calls of the form `new("interval", ...)`, but typically they are built via `interval()`.

### Slots

`.Data`: numeric vector of length two, specifying the interval ranges.

`open`: `logical` vector of length two, specifying if the interval is open or closed on the left and right, respectively.

### Extends

Class "interval" extends "numeric", from data part, and "maybeInterval", directly.

### Methods

`"%in%"` signature(`x = "numeric"`, `table = "interval"`): check if `x` is inside the interval, carefully differentiating open and closed intervals.

`format` signature(`x = "interval"`): ...

`show` signature(`object = "interval"`): ...

`Summary` signature(`x = "interval"`): Group methods, notably `range()`, `min()`, etc.

### Note

There are more sophisticated interval classes, functions and methods, notably in package **intervals**. We only use this as a simple interface in order to specify our copula functions consistently.

### See Also

`interval` constructs "interval" objects conveniently.

### Examples

```
-1:2 %in% interval("(0, Inf)")
## 0 is *not* inside
```



## Description

The *Kendall distribution* of an Archimedean copula is defined by

$$K(u) = P(C(U_1, U_2, \dots, U_d) \leq u),$$

where  $u \in [0, 1]$ , and the  $d$ -dimensional  $(U_1, U_2, \dots, U_d)$  is distributed according to the copula  $C$ . Note that the random variable  $C(U_1, U_2, \dots, U_d)$  is known as “probability integral transform”. Its distribution function  $K$  is equal to the identity if  $d = 1$ , but is non-trivial for  $d \geq 2$ .

`Kn()` computes the empirical Kendall distribution function, `pK()` the distribution function (so  $K()$  itself), `qK()` the quantile function, `dK()` the density, and `rK()` random number generation from  $K()$  for an Archimedean copula.

## Usage

```
Kn(u, x, method = c("GR", "GNZ")) # empirical Kendall distribution function
dK(u, copula, d, n.MC = 0, log.p = FALSE) # density
pK(u, copula, d, n.MC = 0, log.p = FALSE) # df
qK(p, copula, d, n.MC = 0, log.p = FALSE, # quantile function
    method = c("default", "simple", "sort", "discrete", "monoH.FC"),
    u.grid, xtraChecks = FALSE, ...)
rK(n, copula, d) # random number generation
```

## Arguments

|                     |                                                                                                                                                                                                                                                                                                                                                                                      |
|---------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>u</code>      | evaluation point(s) (in $[0, 1]$ ).                                                                                                                                                                                                                                                                                                                                                  |
| <code>x</code>      | data (in the $d$ -dimensional space) based on which the Kendall distribution function is estimated.                                                                                                                                                                                                                                                                                  |
| <code>copula</code> | <a href="#">acopula</a> with specified parameter, or (currently for <code>rK</code> only) a <a href="#">outer_nacopula</a> .                                                                                                                                                                                                                                                         |
| <code>d</code>      | dimension (not used when <code>copula</code> is an <a href="#">outer_nacopula</a> ).                                                                                                                                                                                                                                                                                                 |
| <code>n.MC</code>   | <a href="#">integer</a> , if positive, a Monte Carlo approach is applied with sample size equal to <code>n.MC</code> to evaluate the generator derivatives involved; otherwise ( <code>n.MC = 0</code> ) the exact formula is used based on the generator derivatives as found by Hofert et al. (2012).                                                                              |
| <code>log.p</code>  | <a href="#">logical</a> ; if TRUE, probabilities $p$ are given as $\log p$ .                                                                                                                                                                                                                                                                                                         |
| <code>p</code>      | probabilities or log-probabilities if <code>log.p</code> is true.                                                                                                                                                                                                                                                                                                                    |
| <code>method</code> | for <code>qK()</code> , character string for the method how to compute the quantile function of $K$ ; available are:<br><b>"default"</b> default method. Currently chooses <code>method="monoH.FC"</code> with <code>u.grid = 0:128/128</code> . This is fast but not too accurate (see example).<br><b>"simple"</b> straightforward root finding based on <a href="#">uniroot</a> . |

"**sort**" root finding based on `uniroot` but first sorting `u`.

"**discrete**" first,  $K$  is evaluated at the given grid points `u.grid` (which should contain 0 and 1). Based on these probabilities, quantiles are computed with `findInterval`.

"**monoH.FC**" first,  $K$  is evaluated at the given grid points `u.grid`. A monotone spline is then used to approximate  $K$ . Based on this approximation, quantiles are computed with `uniroot`.

For `Kn()`, character string indicating the method according to which the empirical Kendall distribution is computed; available are:

"**GR**" the default. Computed as in Genest and Rivest (1993, Equations (4) and (5)).

"**GNZ**" computed as in Genest et al. (2011, Equation (19) and Lemma 1); this is guaranteed to satisfy that the estimator lies above the diagonal at any point in  $[0,1)$ .

|                         |                                                                                                                                                                                                                                                                                                                                         |
|-------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>u.grid</code>     | (for <code>method="discrete"</code> ): The grid on which $K$ is evaluated, a <code>numeric</code> vector.                                                                                                                                                                                                                               |
| <code>xtraChecks</code> | <i>experimental</i> logical indicating if extra checks should be done before calling <code>uniroot()</code> in some cases.                                                                                                                                                                                                              |
| <code>...</code>        | additional arguments passed to <code>uniroot</code> (for <code>method="default"</code> , <code>method="simple"</code> , <code>method="sort"</code> , and <code>method="monoH.FC"</code> ) or <code>findInterval</code> (for <code>method="discrete"</code> ), notably <code>tol</code> ( <code>uniroot</code> ) for increased accuracy. |
| <code>n</code>          | sample size for <code>rK</code> .                                                                                                                                                                                                                                                                                                       |

## Details

For a completely monotone Archimedean generator  $\psi$ ,

$$K(u) = \sum_{k=0}^{d-1} \frac{\psi^{(k)}(\psi^{-1}(u))}{k!} (-\psi^{-1}(u))^k, \quad u \in [0, 1];$$

see Barbe et al. (1996). The corresponding density is

$$\frac{(-1)^d \psi^{(d)}(\psi^{-1}(u))}{(d-1)!} (-\psi^{-1}(u))' (\psi^{-1}(u))^{d-1}$$

## Value

The empirical Kendall distribution function, density, distribution function, quantile function and random number generator.

## Note

Currently, the "default" method of `qK()` is fast but not very accurate, see the 'Examples' for more accuracy (with more CPU effort).

## References

- Barbe, P., Genest, C., Ghoudi, K., and Rémillard, B. (1996), On Kendall's Process, *Journal of Multivariate Analysis* **58**, 197–229.
- Hofert, M., Mächler, M., and McNeil, A. J. (2012). Likelihood inference for Archimedean copulas in high dimensions under known margins. *Journal of Multivariate Analysis* **110**, 133–150. doi:10.1016/j.jmva.2012.02.019
- Genest, C. and Rivest, L.-P. (1993). Statistical inference procedures for bivariate Archimedean copulas. *Journal of the American Statistical Association* **88**, 1034–1043.
- Genest, C., G. Nešlehová, J., and Ziegel, J. (2011). Inference in multivariate Archimedean copula models. *TEST* **20**, 223–256.

## See Also

[htrafo](#) or [emde](#) (where **K** is used); [splinefun](#)(\*, "monoHC") for that method.

## Examples

```
tau <- 0.5
(theta <- copGumbel@iTau(tau)) # 2
d <- 20
(cop <- onacopulaL("Gumbel", list(theta,1:d)))

## Basic check of the empirical Kendall distribution function
set.seed(271)
n <- 1000
U <- rCopula(n, copula = cop)
X <- qnorm(U)
K.sample <- pCopula(U, copula = cop)
u <- seq(0, 1, length.out = 256)
edfK <- ecdf(K.sample)
plot(u, edfK(u), type = "l", ylim = 0:1,
      xlab = quote(italic(u)), ylab = quote(K[n](italic(u)))) # simulated
K.n <- Kn(u, x = X)
lines(u, K.n, col = "royalblue3") # Kn
## Difference at 0
edfK(0) # edf of K at 0
K.n[1] # K_n(0); this is > 0 since K.n is the edf of a discrete distribution
## => therefore, Kn(K.sample, x = X) is not uniform
plot(Kn(K.sample, x = X), ylim = 0:1)
## Note: Kn(0) -> 0 for n -> Inf

## Compute Kendall distribution function
u <- seq(0,1, length.out = 255)
Ku <- pK(u, copula = cop@copula, d = d) # exact
Ku.MC <- pK(u, copula = cop@copula, d = d, n.MC = 1000) # via Monte Carlo
stopifnot(all.equal(log(Ku),
                    pK(u, copula = cop@copula, d = d, log.p=TRUE)))# rel.err 3.2e-16

## Build sample from K
set.seed(1)
```

```

n <- 200
W <- rK(n, copula = cop)

## Plot empirical distribution function based on W
## and the corresponding theoretical Kendall distribution function
## (exact and via Monte Carlo)
plot(ecdf(W), col = "blue", xlim = 0:1, verticals=TRUE,
     main = quote("Empirical"~ F[n](C(U)) ~
                  "and its Kendall distribution" ~ K(u)),
     do.points = FALSE, asp = 1)
abline(0,1, lty = 2); abline(h = 0:1, v = 0:1, lty = 3, col = "gray")
lines(u, Ku.MC, col = "red") # not quite monotone
lines(u, Ku, col = "black") # strictly monotone:
stopifnot(diff(Ku) >= 0)
legend(.25, .75, expression(F[n], K[MC](u), K(u)),
      col=c("blue" , "red", "black"), lty = 1, lwd = 1.5, bty = "n")

if(require("Rmpfr")) { # pK() now also works with high precision numbers:
  uM <- mpfr(0:255, 99)/256
  if(FALSE) {
    # not yet, now fails in polyG() :
    KuM <- pK(uM, copula = cop@copula, d = d)
    ## debug(copula:::pK)
    debug(copula:::polyG)
  }
}# if( Rmpfr )

## Testing qK
pexpr <- quote( 0:63/63 ); p <- eval(pexpr)
d <- 10
cop <- onacopula("Gumbel", list(theta = 2, 1:d))
system.time(qK0 <- qK(p, copula = cop@copula, d = d)) # "default" - fast

system.time(qK1 <- qK(p, copula= cop@copula, d=d, method = "simple"))
system.time(qK1. <- qK(p, copula= cop@copula, d=d, method = "simple", tol = 1e-12))
system.time(qK2 <- qK(p, copula= cop@copula, d=d, method = "sort"))
system.time(qK2. <- qK(p, copula= cop@copula, d=d, method = "sort", tol = 1e-12))
system.time(qK3 <- qK(p, copula= cop@copula, d=d, method = "discrete", u.grid = 0:1e4/1e4))
system.time(qK4 <- qK(p, copula= cop@copula, d=d, method = "monoH.FC",
  u.grid = 0:5e2/5e2))
system.time(qK4. <- qK(p, copula= cop@copula, d=d, method = "monoH.FC",
  u.grid = 0:5e2/5e2, tol = 1e-12))
system.time(qK5 <- qK(p, copula= cop@copula, d=d, method = "monoH.FC",
  u.grid = 0:5e3/5e3))
system.time(qK5. <- qK(p, copula= cop@copula, d=d, method = "monoH.FC",
  u.grid = 0:5e3/5e3, tol = 1e-12))
system.time(qK6 <- qK(p, copula= cop@copula, d=d, method = "monoH.FC",
  u.grid = (0:5e3/5e3)^2))
system.time(qK6. <- qK(p, copula= cop@copula, d=d, method = "monoH.FC",
  u.grid = (0:5e3/5e3)^2, tol = 1e-12))

```

```

## Visually they all coincide :
cols <- adjustcolor(c("gray50", "gray80", "light blue",
                    "royal blue", "purple3", "purple4", "purple"), 0.6)
matplot(p, cbind(qK0, qK1, qK2, qK3, qK4, qK5, qK6), type = "l", lwd = 2*7:1, lty = 1:7, col = cols,
        xlab = bquote(p == .(pexpr)), ylab = quote({K^{-1}}(u)),
        main = "qK(p, method = *)")
legend("topleft", col = cols, lwd = 2*7:1, lty = 1:7, bty = "n", inset = .03,
      legend= paste0("method= ",
                    sQuote(c("default", "simple", "sort",
                              "discrete(1e4)", "monoH.FC(500)", "monoH.FC(5e3)", "monoH.FC(*^2)"))))

## See they *are* inverses (but only approximately!):
eqInv <- function(qK) all.equal(p, pK(qK, cop@copula, d=d), tol=0)

eqInv(qK0 ) # "default"          0.03  worst
eqInv(qK1 ) # "simple"           0.0011 - best
eqInv(qK1.) # "simple", e-12     0.00000 (8.73 e-13) !
eqInv(qK2 ) # "sort"           0.0013 (close)
eqInv(qK2.) # "sort", e-12     0.00000 (7.32 e-12)
eqInv(qK3 ) # "discrete"       0.0026
eqInv(qK4 ) # "monoH.FC(500)"   0.0095
eqInv(qK4.) # "m.H.FC(5c)e-12" 0.00963
eqInv(qK5 ) # "monoH.FC(5e3)"   0.001148
eqInv(qK5.) # "m.H.FC(5k)e-12" 0.000989
eqInv(qK6 ) # "monoH.FC(*^2)"  0.001111
eqInv(qK6.) # "m.H.FC(*^2)e-12"0.00000 (1.190 e-09)

## and ensure the differences are not too large
stopifnot(
  all.equal(qK0, qK1, tol = 1e-2) # !
  ,
  all.equal(qK1, qK2, tol = 1e-4)
  ,
  all.equal(qK2, qK3, tol = 1e-3)
  ,
  all.equal(qK3, qK4, tol = 1e-3)
  ,
  all.equal(qK4, qK0, tol = 1e-2) # !
)

stopifnot(all.equal(p, pK(qK0, cop@copula, d=d), tol = 0.04))

```

---

## Description

Creates an object representing a copula constructed using *Khoudraji's device* (Khoudraji, 1995). The resulting R object is either of class "[khoudrajiBivCopula](#)", "[khoudrajiExplicitCopula](#)" or "[khoudrajiCopula](#)".

In the bivariate case, given two copulas  $C_1$  and  $C_2$ , Khoudraji's device consists of defining a copula whose c.d.f. is given by:

$$C_1(u_1^{1-a_1}, u_2^{1-a_2})C_2(u_1^{a_1}, u_2^{a_2})$$

where  $a_1$  and  $a_2$  are *shape parameters* in  $[0,1]$ .

The construction principle (see also Genest et al. 1998) is a special case of that considered in Liebscher (2008).

### Usage

```
khoudrajiCopula(copula1 = indepCopula(), copula2 = indepCopula(dim = d),
               shapes = rep(NA_real_, dim(copula1)))
```

### Arguments

copula1, copula2

each a `copula` (possibly generalized, e.g., also a "`rotCopula`") of the same dimension  $d$ . By default independence copulas, where copula2 gets the dimension from copula1.

shapes `numeric` vector of length  $d$ , with values in  $[0, 1]$ .

### Details

If the argument copulas are bivariate, an object of class "`khoudrajiBivCopula`" will be constructed. If they are exchangeable and  $d$ -dimensional with  $d > 2$ , and if they have explicit p.d.f. and c.d.f. expressions, an object of class "`khoudrajiExplicitCopula`" will be constructed. For the latter two classes, density evaluation is implemented, and fitting and goodness-of-fit testing can be attempted. If  $d > 2$  but one of the argument copulas does not have explicit p.d.f. and c.d.f. expressions, or is not exchangeable, an object of class "`khoudrajiCopula`" will be constructed, for which density evaluation is not possible.

### Value

A new object of class "`khoudrajiBivCopula`" in dimension two or of class "`khoudrajiExplicitCopula`" or "`khoudrajiCopula`" when  $d > 2$ .

### References

Genest, C., Ghoudi, K., and Rivest, L.-P. (1998), Discussion of "Understanding relationships using copulas", by Frees, E., and Valdez, E., *North American Actuarial Journal* **3**, 143–149.

Khoudraji, A. (1995), Contributions à l'étude des copules et à la modélisation des valeurs extrêmes bivariées, *PhD thesis, Université Laval, Québec, Canada*.

Liebscher, E. (2008), Construction of asymmetric multivariate copulas, *Journal of Multivariate Analysis* **99**, 2234–2250.

**Examples**

```

## A bivariate Khoudraji-Clayton copula
kc <- khoudrajiCopula(copula2 = claytonCopula(6),
                    shapes = c(0.4, 0.95))
class(kc) # "kh..._Biv_Copula"
kc
contour(kc, dCopula, nlevels = 20, main = "dCopula(<khoudrajiBivCopula>)")

## A Khoudraji-Clayton copula with second shape parameter fixed
kcf <- khoudrajiCopula(copula2 = claytonCopula(6),
                    shapes = fixParam(c(0.4, 0.95), c(FALSE, TRUE)))
kcf. <- setTheta(kcf, c(3, 0.2)) # (change *free* param's only)
validObject(kcf) & validObject(kcf.)

## A "nested" Khoudraji bivariate copula
kgkcf <- khoudrajiCopula(copula1 = gumbelCopula(3),
                    copula2 = kcf,
                    shapes = c(0.7, 0.25))
kgkcf # -> 6 parameters (1 of 6 is 'fixed')
contour(kgkcf, dCopula, nlevels = 20,
        main = "dCopula(<khoudrajiBivC.(nested)>)")

(Xtras <- copula:::doExtras()) # determine whether examples will be extra (long)
n <- if(Xtras) 300 else 64 # sample size (realistic vs short for example)

u <- rCopula(n, kc)
plot(u)

## For likelihood (or fitting), specify the "free" (non-fixed) param's:
##          C1: C2c C2s1 sh1 sh2
loglikCopula(c(3, 6, 0.4, 0.7, 0.25),
            u = u, copula = kgkcf)

## Fitting takes time (using numerical differentiation) and may be difficult:

## Starting values are required for all parameters
f.IC <- fitCopula(khoudrajiCopula(copula2 = claytonCopula()),
                start = c(1.1, 0.5, 0.5), data = pobs(u),
                optim.method = "Nelder-Mead")
summary(f.IC)
confint(f.IC) # (only interesting for reasonable sample size)

## Because of time, don't run these by default :

## Second shape parameter fixed to 0.95
kcf2 <- khoudrajiCopula(copula2 = claytonCopula(),
                    shapes = fixParam(c(NA_real_, 0.95), c(FALSE, TRUE)))
system.time(
f.ICf <- fitCopula(kcf2, start = c(1.1, 0.5), data = pobs(u),
                optim.method = "Nelder-Mead")
) # ~ 7-8 sec
confint(f.ICf) # !

```

```

coef(f.ICf, SE=TRUE)

## With a different optimization method
system.time(
f.IC2 <- fitCopula(kcf2, start = c(1.1, 0.5), data = pobs(u),
                  optim.method = "BFGS")
)
printCoefmat(coef(f.IC2, SE=TRUE), digits = 3) # w/o unuseful extra digits

if(Xtras >= 2) { # really S..L..O..W... -----

## GOF example
optim.method <- "Nelder-Mead" #try "BFGS" as well
gofCopula(kcf2, x = u, start = c(1.1, 0.5), optim.method = optim.method)
gofCopula(kcf2, x = u, start = c(1.1, 0.5), optim.method = optim.method,
          sim = "mult")
## The goodness-of-fit tests should hold their level
## but this would need to be tested

## Another example under the alternative
u <- rCopula(n, gumbelCopula(4))
gofCopula(kcf2, x = u, start = c(1.1, 0.5), optim.method = optim.method)
gofCopula(kcf2, x = u, start = c(1.1, 0.5), optim.method = optim.method,
          sim = "mult")

}## ----- end { really slow gofC*() } -----

## Higher-dimensional constructions

## A three dimensional Khoudraji-Clayton copula
kcd3 <- khoudrajiCopula(copula1 = indepCopula(dim=3),
                      copula2 = claytonCopula(6, dim=3),
                      shapes = c(0.4, 0.95, 0.95))

n <- if(Xtras) 1000 else 100 # sample size (realistic vs short for example)
u <- rCopula(n, kcd3)
splom2(u)
v <- matrix(runif(15), 5, 3)
dCopula(v, kcd3)

## A four dimensional Khoudraji-Normal copula
knd4 <- khoudrajiCopula(copula1 = indepCopula(dim=4),
                      copula2 = normalCopula(.9, dim=4),
                      shapes = c(0.4, 0.95, 0.95, 0.95))

knd4
stopifnot(class(knd4) == "khourajicopula")
u <- rCopula(n, knd4)
splom2(u)
## TODO :
## dCopula(v, knd4) ## not implemented

```



---

khourajCopula-class *Class "khourajCopula" and its Subclasses*

---

### Description

The *virtual* class "asymCopula" of (conceptually) all asymmetric copulas and its 'subclass' "asym2copula" of those which are constructed from two copulas.

More specifically, the class "khourajCopula" and its two subclasses "khourajBivCopula" and "khourajExplicitCopula" represent copulas constructed using Khouraji's device from two copulas of the same dimension; see [khourajCopula\(\)](#) for more details.

### Objects from the Class

Objects are typically created via [khourajCopula\(...\)](#).

### Slots

As these classes extend "copula", they have all its slots: `dimension`, `parameters`, `param.names`, `param.lowbnd`, `param.upbnd`, and `fullname`. The classes "khourajCopula" and "khourajBivCopula" have the extra slots

`copula1`: object of class "copula".

`copula2`: second object of class "copula".

In addition to these, the class "khourajExplicitCopula" has the slots

`exprdist`: an [expression](#), ...

`derExprs1`: an [expression](#) of length  $d$ , ...

`derExprs2`: an [expression](#) of length  $d$ , ...

### Methods

When possible, methods are defined at the "khourajCopula" class level. The implementation of method [dCopula](#) for instance is however not possible at that level. In addition, it differs for "khourajBivCopula" and "khourajExplicitCopula" classes.

### References

Genest, C., Ghoudi, K., and Rivest, L.-P. (1998), Discussion of "Understanding relationships using copulas", by Frees, E., and Valdez, E., *North American Actuarial Journal* **3**, 143–149.

Khouraji, A. (1995), Contributions à l'étude des copules et à la modélisation des valeurs extrêmes bivariées, *PhD thesis, Université Laval, Québec, Canada*.

Liebscher, E. (2008), Construction of asymmetric multivariate copulas, *Journal of Multivariate Analysis* **99**, 2234–2250.

**See Also**

[khourajCopula\(\)](#)

**Examples**

```
showClass("khourajCopula")# two subclasses

## all methods applicable to these subclasses:
(meths <- sapply(names(getClass("khourajCopula")@subclasses),
                 function(CL) methods(class = CL),
                 simplify=FALSE))
```

---

log1mexp

*Compute  $f(a) = \log(1 \pm \exp(-a))$  Numerically Optimally*

---

**Description**

Compute  $f(a) = \log(1 - \exp(-a))$ , respectively  $g(x) = \log(1 + \exp(x))$  quickly numerically accurately.

**Usage**

```
log1mexp(a, cutoff = log(2))
log1pexp(x, c0 = -37, c1 = 18, c2 = 33.3)
```

**Arguments**

**a** numeric vector of positive values

**x** numeric vector

**cutoff** positive number;  $\log(2)$  is “optimal”, but the exact value is unimportant, and anything in  $[0.5, 1]$  is fine.

**c0, c1, c2** cutoffs for log1pexp; see below.

**Value**

$f(a) == \log(1 - \exp(-a)) == \log1p(-\exp(-a)) == \log(-\expm1(-a))$   
 or  
 $g(x) == \log(1 + \exp(x)) == \log1p(\exp(x))$   
 computed accurately and quickly

**References**

Martin Mächler (2012). Accurately Computing  $\log(1 - \exp(-|a|))$ ; <https://CRAN.R-project.org/package=Rmpfr/vignettes/log1mexp-note.pdf>.

**Examples**

```

a <- 2^seq(-58,10, length = 256)
fExpr <- expression(
  log(1 - exp(-a)),
  log(-expm1(-a)),
  log1p(-exp(-a)),
  log1mexp(a))
names(fExpr) <- c("DEF", "expm1", "log1p", "F")
str(fa <- do.call(cbind, as.list(fExpr)))
head(fa)# expm1() works here
tail(fa)# log1p() works here

## graphically:
lwd <- 1.5*(5:2); col <- adjustcolor(1:4, 0.4)
op <- par(mfcol=c(1,2), mgp = c(1.25, .6, 0), mar = .1+c(3,2,1,1))
  matplot(a, fa, type = "l", log = "x", col=col, lwd=lwd)
  legend("topleft", fExpr, col=col, lwd=lwd, lty=1:4, bty="n")
  # expm1() & log1mexp() work here

  matplot(a, -fa, type = "l", log = "xy", col=col, lwd=lwd)
  legend("left", paste("-",fExpr), col=col, lwd=lwd, lty=1:4, bty="n")
  # log1p() & log1mexp() work here
par(op)

curve(log1pexp, -10, 10, asp=1)
abline(0,1, h=0,v=0, lty=3, col="gray")

## Cutoff c1 for log1pexp() -- not often "needed":
curve(log1p(exp(x)) - log1pexp(x), 16, 20, n=2049)
## need for *some* cutoff:
x <- seq(700, 720, by=2)
cbind(x, log1p(exp(x)), log1pexp(x))

## Cutoff c2 for log1pexp():
curve((x+exp(-x)) - x, 20, 40, n=1025)
curve((x+exp(-x)) - x, 33.1, 33.5, n=1025)

```

---

loss

*LOSS and ALAE Insurance Data*


---

**Description**

Indemnity payment and allocated loss adjustment expense from an insurance company.

**Usage**

```
data(loss, package="copula")
```

**Format**

A data frame with 1500 observations of the following 4 variables:

`loss` a numeric vector of loss amount up to the `limit`.

`alae` a numeric vector of the corresponding allocated loss adjustment expense.

`limit` a numeric vector of limit (-99 means no limit).

`censored` 1 means censored (limit reached) and 0 otherwise.

**References**

Frees, E. and Valdez, E. (1998). Understanding relationships using copulas. *North American Actuarial Journal* **2**, 1–25.

**Examples**

```
data(loss)
```

---

margCopula

*Marginal copula of a Copula With Specified Margins*

---

**Description**

The marginal copula of a copula  $C(u_1, \dots, u_d)$  is simply the restriction of  $C$  on a subset of the the coordinate (directions)  $u_1, \dots, u_d$ .

**Usage**

```
margCopula(copula, keep)
```

**Arguments**

`copula` a "`copula`" (R object) of dimension,  $d$ , say.

`keep` logical vector (of length  $d$ ) indicating which margins to keep.

**Details**

The marginal copula of a copula is needed in practical data analysis when one or more of the components of some multivariate observations is missing. For normal/t/Archimedean copulas, the marginal copulas can be easily obtained. For a general copula, this may not be an easy problem.

The current implementation only supports normal/t/Archimedean copulas. `margCopula` is generic function with methods for the different copula classes.

**Value**

The marginal copula of the specified margin(s).

**Examples**

```
tc <- tCopula(8:2 / 10, dim = 8, dispstr = "toep")
margCopula(tc, c(TRUE, TRUE, FALSE, TRUE, FALSE, FALSE, TRUE, FALSE))

nc <- normalCopula(.8, dim = 8, dispstr = "ar1")
mnc <- margCopula(nc, c(TRUE, TRUE, FALSE, TRUE, FALSE, FALSE, TRUE, FALSE))
mnc7 <- margCopula(nc, (1:8) != 1)
stopifnot(dim(nc) == 8, dim(mnc) == 4, dim(mnc7) == 7)

gc <- gumbelCopula(2, dim = 8)
margCopula(gc, c(TRUE, TRUE, TRUE, FALSE, FALSE, FALSE, FALSE, FALSE))
```

math-fun

*Sinc, Zolotarev's, and Other Mathematical Utility Functions***Description**

`sinc(x)` computes the *sinc function*  $s(x) = \sin(x)/x$  for  $x \neq 0$  and  $s(0) = 1$ , such that  $s(\cdot)$  is continuous, also at  $x = 0$ .

`A.Z(x, a)` computes Zolotarev's function to the power  $1-a$ .

**Usage**

```
sinc(x)
A.Z(x, alpha, I.alpha = 1 - alpha)
```

**Arguments**

`x` **numeric** argument in  $[0, \pi]$ , typically a vector.  
`alpha` parameter in  $(0,1]$ .  
`I.alpha` must be  $= 1 - \alpha$ , maybe more accurately when  $\alpha$  is very close to 1.

**Details**

For more details about Zolotarev's function, see, for example, Devroye (2009).

**Value**

`A.Z(x, alpha)` is  $\tilde{A}_Z(x, \alpha)$ , defined as

$$\frac{\sin(\alpha x)^\alpha \sin((1 - \alpha)x)^{1-\alpha}}{\sin(x)}, \quad x \in [0, \pi],$$

where  $\alpha \in (0, 1]$  is `alpha`.

## References

Devroye, L. (2009) Random variate generation for exponentially and polynomially tilted stable distributions, *ACM Transactions on Modeling and Computer Simulation* **19**, 18, 1–20.

## See Also

`retstable` internally makes use of these functions.

## Examples

```
curve(sinc, -15,25); abline(h=0,v=0, lty=2)
curve(A..Z(x, 0.25), xlim = c(-4,4),
      main = "Zolotarev's function A(x) ^ 1-alpha")
```

---

matrix\_tools

*Tools to Work with Matrices*

---

## Description

`p2P()` creates a **matrix** from a given **vector** of parameters. `P2p()` creates a numeric vector from a given **matrix**, currently useful for elliptical copulas.

`getSigma()` returns the  $d \times d$  symmetric matrix  $\Sigma$  which is called “Rho” as well, written (capital Greek  $\rho$  !) as  $P$  (and hence sometimes erroneously pronounced “Pee”). Note that `getSigma()` works for all elliptical copulas and uses `p2P()` for the “unstructured” case, `dispstr = “un”`.

`extremePairs()` identifies pairs with the largest (or smallest or both) entries in a symmetric matrix.

## Usage

```
p2P(param, d = floor(1 + sqrt(2*length(param))))
P2p(P)
getSigma(copula)
extremePairs(x, n = 6, method = c("largest", "smallest", "both"),
            use.names = FALSE)
```

## Arguments

|                     |                                                                                                                                                                                                                                                                                      |
|---------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>param</code>  | a parameter vector.                                                                                                                                                                                                                                                                  |
| <code>d</code>      | dimension of the resulting <b>matrix</b> . The default is correct under the assumption (of <code>p2P()</code> in general!) that <code>param</code> is the lower-triangular part of a correlation matrix $P$ and hence corresponds to <code>ellipCopula(..., dispstr = “un”)</code> . |
| <code>P</code>      | a <b>matrix</b> which should be converted to a vector.                                                                                                                                                                                                                               |
| <code>copula</code> | an <b>elliptical</b> copula, i.e., an object (extending) class <code>ellipCopula</code> ; typically resulting from <code>tCopula()</code> or <code>normalCopula()</code> .                                                                                                           |
| <code>x</code>      | a symmetric <b>matrix</b> .                                                                                                                                                                                                                                                          |
| <code>n</code>      | the number of pairs with smallest (or largest) values to be displayed.                                                                                                                                                                                                               |

|           |                                                                                                                                                                                                                                                                                                                                                                                         |
|-----------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| method    | a <a href="#">character</a> string indicating the method to be used (with "largest" to compute the n pairs with largest entries in x (sorted in decreasing order); with "smallest" to compute the n pairs with smallest entries in x (sorted in increasing order); and with "both" to compute the 2n pairs with n largest entries and n smallest entries (sorted in decreasing order)). |
| use.names | A <a href="#">logical</a> indicating whether colnames(x) are used as labels (if !is.null(colnames(x))).                                                                                                                                                                                                                                                                                 |

### Details

These auxiliary functions are often used when working with elliptical copulas.

### Value

p2P: a symmetric [matrix](#) with ones on the diagonal and the values of param filled column-wise below the diagonal (which corresponds to row-wise filling above the diagonal).

P2p: [vector](#) of column-wise below-diagonal entries of P (equal to the row-wise above-diagonal entries in case of a symmetric matrix).

getSigma: [matrix](#) as from p2P() for all cases of elliptical copulas.

extremePairs: a [data.frame](#) consisting of three columns (row (index or name), col (index or name), value).

### See Also

[ellipCopula](#), [tCopula](#), [normalCopula](#).

### Examples

```
## display the two simple definitions:
p2P
P2p

param <- (2:7)/10
tC <- tCopula(param, dim = 4, dispstr = "un", df = 3)
## consistency of the three functions :
P <- p2P(param) # (using the default 'd')
stopifnot(identical(param, P2p(P)),
           identical(P, getSigma(tC)))

## Toeplitz case:
(tCt <- tCopula((2:6)/10, dim = 6, disp = "toep"))
(rhoP <- tCt@getRho(tCt))
stopifnot(identical(getSigma(tCt),
                    toeplitz(c(1, rhoP))))

## "AR1" case:
nC.7 <- normalCopula(0.8, dim = 7, dispstr = "ar1")
(Sar1.7 <- getSigma(nC.7))
0.8^(0:(7-1)) # 1 0.8 0.64 0.512 ..
stopifnot(all.equal(Sar1.7, toeplitz(0.8^(0:(7-1)))))
```

---

 mixCopula

 Create Mixture of Copulas
 

---

### Description

A mixture of  $m$  copulas of dimension  $d$  with weights  $w_j, j = 1, 2, \dots, m$  is itself a  $d$ -dimensional copula, with cumulative distribution function

$$C(x) = \sum_{j=1}^m w_j C_j(x),$$

and (probability) density function

$$c(x) = \sum_{j=1}^m w_j c_j(x),$$

where  $C_j$  are the CDFs and  $c_j$  are the densities of the  $m$  component copulas,  $j = 1, 2, \dots, m$ .

### Usage

```
mixCopula(coplist, w = NULL)
```

### Arguments

|         |                                                                                                                                     |
|---------|-------------------------------------------------------------------------------------------------------------------------------------|
| coplist | a <a href="#">list</a> of length $m (\geq 1)$ copulas (each inheriting from <a href="#">parCopula</a> ), all of the same dimension. |
| w       | numeric vector of length $m$ of non-negative mixture weights, or NULL, which means equal weights.                                   |

### Details

It easy to see that the tail dependencies [lambda\(\)](#) and Spearman's rank correlation [rho\(\)](#) can be computed as mixture of the individual measures.

### Value

an object of [class mixCopula](#)

### See Also

[khourajCopula](#), [rotCopula](#) also create new copula models from existing ones.



## Examples

```

mC <- mixCopula(list(gumbelCopula(2.5, dim=3),
                    claytonCopula(pi, dim=3),
                    tCopula(0.7, dim=3)),
                c(2,2,4)/8)

mC
stopifnot(dim(mC) == 3)

set.seed(17)
uM <- rCopula(600, mC)
splom2(uM, main = "mixCopula( gumbel, clayton, t-Cop) ")
d.uM <- dCopula(uM, mC)
p.uM <- pCopula(uM, mC)

## mix a Gumbel with a rotated Gumbel (with equal weights 1/2):
mGG <- mixCopula(list(gumbelCopula(2), rotCopula(gumbelCopula(1.5))))
rho(mGG) # 0.57886
lambda(mGG)# both lower and upper tail dependency

loglikCopula(c(2.5, pi, rho.1=0.7, df = 4, w = c(2,2,4)/8),
             u = uM, copula = mC)
## define (profiled) log-likelihood function of two arguments (df, rho) :
ll.df <- Vectorize(function(df, rho)
                  loglikCopula(c(2.5, pi, rho.1=rho, df=df, w = c(2,2,4)/8),
                              uM, mC))
(df. <- 1/rev(seq(1/8, 1/2, length=21)))# in [2, 8] equidistant in 1/. scale

ll. <- ll.df(df., rho = (rh1 <- 0.7))
plot(df., ll., type = "b", main = "loglikCopula(.,.,rho = 0.7, df, ..), u, <mixCopula>")

if(!exists("Xtras")) Xtras <- copula::doExtras() ; cat("Xtras: ", Xtras, "\n")
if (Xtras) withAutoprint({
  Rhos <- seq(0.55, 0.70, by = 0.01)
  ll.m <- matrix(NA, nrow=length(df.), ncol=length(Rhos))
  for(k in seq_along(Rhos)) ll.m[,k] <- ll.df(df., rho = Rhos[k])
  tit <- "loglikelihood(<tCop>, true param. for rest)"
  persp      (df., Rhos, ll.m, phi=30, theta = 50, ticktype="detailed", main = tit)
  filled.contour(df., Rhos, ll.m, xlab="df", ylab = "rho", main = tit)
})

## fitCopula() example -----

## 1) with "fixed" weights :

(mNt <- mixCopula(list(normalCopula(0.95), tCopula(-0.7)), w = c(1, 2) / 3))
set.seed(1452) ; U <- pobs(rCopula(1000, mNt))
(m1 <- mixCopula(list(normalCopula(), tCopula()), w = mNt@w))

getTheta(m1, freeOnly = TRUE, attr = TRUE)
getTheta(m1, named=TRUE)
isFree(m1) # all of them; --> now fix the weights :
fixedParam(m1) <- fx <- c(FALSE, FALSE, FALSE, TRUE, TRUE)

```

```

stopifnot(identical(isFree(m1), !fx))

if(Xtras) withAutoprint({ ## time
  system.time( # ~ 16 sec (nb-mm4) :
    fit <- fitCopula(m1, start = c(0, 0, 10), data = U)
  )
  fit
  summary(fit) #-> incl 'Std.Error' (which seems small for rho1 !)
})

## 2) with "free" weights (possible since copula 1.0-0):

(mNt2 <- mixCopula(list(normalCopula(0.9), tCopula(-0.8)), w = c(1, 3) / 4))
set.seed(1959) ; U2 <- pobs(rCopula(2000, mNt2))
if(Xtras) withAutoprint({ ## time
  m2 <- mixCopula(list(normalCopula(), tCopula()), w = mNt@w)
  system.time( # ~ 13.5 sec (lynne) :
    f2 <- fitCopula(m2, start = c(0, 0, 10, c(1/2, 1/2)), data = U2)
  )
  f2
  summary(f2) # NA for 'Std. Error' as did *not* estimate.variance
  summary(f2, orig=FALSE) # default 'orig=TRUE': w-scale; whereas
  coef(f2, orig=FALSE) # 'orig=FALSE' => shows 'l-scale' instead
})

```

---

mixCopula-class

Class "mixCopula" of Copula Mixtures

---

### Description

The class "mixCopula" is the class of all finite mixtures of copulas.

These are given by (a list of)  $m$  "arbitrary" copulas, and their respective  $m$  non-negative probability weights.

### Objects from the Class

Objects are typically created by `mixCopula()`.

### Slots

**w:** Object of class "mixWeights", basically a non-negative `numeric` vector of length, say  $m$ , which sums to one.

**cops:** Object of class "parClist", a `list` of (parametrized) copulas, "parCopula".

### Extends

Class "parCopula", directly. Class "Copula", by class "parCopula", distance 2.

**Methods**

**dim** signature(x = "mixCopula"): dimension of copula.

**rho** signature(x = "mixCopula"): Spearman's rho of copula x.

**lambda** signature(x = "mixCopula"): lower and upper tail dependencies `lambda`,  $(\lambda[L], \lambda[U])$ , of the mixture copula.

**Note**

As the probability weights must sum to one (1), which is part of the validity (see `validObject`) of an object of class "mixWeights", the number of "free" parameters inherently is (at most) one *less* than the number of mixture components *m*.

Because of that, it does not make sense to fix (see `fixParam` or `fixedParam<-`) all but one of the weights: Either all are fixed, or at least two must be free. Further note, that the definition of free or fixed parameters, and the meaning of the methods (for `mixCopula`) of `getTheta`, `setTheta` and `fixedParam<-` will probably change in a next release of package **copula**, where it is planned to use a reparametrization better suited for `fitCopula`.

**See Also**

`mixCopula` for creation and examples.

**Examples**

```
showClass("mixCopula")
```

---

moCopula

*The Marshall-Olkin Copula*

---

**Description**

Computes Marshall-Olkin copulas in the bivariate case.

**Usage**

```
moCopula(param = NA_real_, dim = 2L)
```

**Arguments**

`param` **numeric** vector of length two specifying the copula parameters (in  $[0, 1]$ ).

`dim` the dimension of the copula.

**Value**

`moCopula()` is the constructor for objects of class `moCopula`.

**Note**

Marshall-Olkin copulas are only implemented for  $\text{dim} = 2L$ .

**See Also**

The "[moCopula](#)" class, its mathematical definition, etc.

**Examples**

```
alpha <- c(0.2, 0.7)
MO <- moCopula(alpha)
tau(MO) # 0.18
lambda(MO)
stopifnot(all.equal(lambda(MO),
                    c(lower = 0, upper = 0.2)))
wireframe2 (MO, FUN = pCopula) # if you look carefully, you can see the kink
contourplot2(MO, FUN = pCopula)
set.seed(271)
plot(rCopula(1000, MO))
```

---

moCopula-class

*Class "moCopula" of Marshall-Olkin Copulas*


---

**Description**

The Marshall-Olkin copula class.

The (2-dimensional) "MO" copula with parameter  $\theta \in [0, 1]^2$  is (i.e., its CDF is)

$$C(u_1, u_2) = \min(u_1 * u_2^{1 - \theta_2}, u_1^{1 - \theta_1} * u_2).$$

Consequently, the density is undefined on a curve (in  $[0, 1]^2$ ), namely for the points  $\mathbf{u} = (u_1, u_2)$  where two expressions in the above  $\min(f(u), g(u))$  are equal,  $f(u) = g(u)$ . It is easy to see that that is equivalent to

$$u_1^{\theta_1} = u_2^{\theta_2}.$$

**Objects from the Class**

Objects can be created by `new("moCopula", ...)` but are typically produced by `moCopula(...)`.

**Slots**

**dimension:** Numeric (scalar), the dimension of the copula.

**exprdist:** a length two [expression](#) with expressions for the CDF and PDF of the copula.

**parameters:** numeric vector of two parameter values in  $[0, 1]$ .

**param.names:** "[character](#)" vector of length two.

**param.lowbnd:** numeric vector of two values in  $[0, 1]$ .

**param.upbnd:** numeric vector of two values in  $[0, 1]$ .

**fullname:** (deprecated; do not use!)

**Methods**

Typical copula methods work, see "[moCopula](#)" and use `methods(class = "moCopula")`.

**Extends**

Class "moCopula" extends class "[copula](#)" directly.

**References**

Nelsen, R. B. (2006), *An introduction to Copulas*, Springer, New York.

**See Also**

[moCopula](#) for constructing them; [copula-class](#).

**Examples**

```
moCopula()@exprdist[["cdf"]] # a simple definition

methods(class = "moCopula")
contourplot2(moCopula(c(.1, .8)), pCopula, main= "moCopula((0.1, 0.8))")

Xmo <- rCopula(5000, moCopula(c(.2, .5)))
try( # gives an error, as there is no density (!):
  loglikCopula(c(.1, .2), Xmo, moCopula())
)

plot(moCopula(c(.9, .2)), n = 10000, xaxs="i", yaxs="i",
     # opaque color (for "density effect"):
     pch = 16, col = adjustcolor("black", 0.3))
```

---

multIndepTest

*Independence Test Among Continuous Random Vectors Based on the Empirical Copula Process*


---

**Description**

Analog of the independence test based on the empirical copula process proposed by Christian Genest and Bruno Rémillard (see [indepTest](#)) for *random vectors*. The main difference comes from the fact that critical values and p-values are obtained through the bootstrap/permutation methodology, since, here, test statistics are not distribution-free.

**Usage**

```
multIndepTest(x, d, m = length(d), N = 1000, alpha = 0.05,
              verbose = interactive())
```

**Arguments**

|         |                                                                                                                                                                     |
|---------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x       | data frame ( <code>data.frame</code> ) or matrix containing realizations (one per line) of the random vectors whose independence is to be tested.                   |
| d       | dimensions of the random vectors whose realizations are given in x. It is required that <code>sum(d) == ncol(x)</code> .                                            |
| m       | maximum cardinality of the subsets of random vectors for which a test statistic is to be computed. It makes sense to consider $m \ll p$ especially when p is large. |
| N       | number of bootstrap/permutation samples.                                                                                                                            |
| alpha   | significance level used in the computation of the critical values for the test statistics.                                                                          |
| verbose | a logical specifying if progress should be displayed via <code>txtProgressBar</code> .                                                                              |

**Details**

See the references below for more details, especially the last one.

**Value**

The function "multIndepTest" returns an object of class "indepTest" whose attributes are: `subsets`, `statistics`, `critical.values`, `pvalues`, `fisher.pvalue` (a p-value resulting from a combination *à la* Fisher of the subset statistic p-values), `tippett.pvalue` (a p-value resulting from a combination *à la* Tippett of the subset statistic p-values), `alpha` (global significance level of the test), `beta` ( $1 - \text{beta}$  is the significance level per statistic), `global.statistic` (value of the global Cramér-von Mises statistic derived directly from the independence empirical copula process - see In in the last reference) and `global.statistic.pvalue` (corresponding p-value).

The former argument `print.every` is deprecated and not supported anymore; use `verbose` instead.

**References**

- Deheuvels, P. (1979). La fonction de dépendance empirique et ses propriétés: un test non paramétrique d'indépendance, *Acad. Roy. Belg. Bull. Cl. Sci.*, 5th Ser. **65**, 274–292.
- Deheuvels, P. (1981), A non parametric test for independence, *Publ. Inst. Statist. Univ. Paris.* **26**, 29–50.
- Genest, C. and Rémillard, B. (2004), Tests of independence and randomness based on the empirical copula process. *Test* **13**, 335–369.
- Genest, C., Quessy, J.-F., and Rémillard, B. (2006). Local efficiency of a Cramer-von Mises test of independence, *Journal of Multivariate Analysis* **97**, 274–294.
- Genest, C., Quessy, J.-F., and Rémillard, B. (2007), Asymptotic local efficiency of Cramér-von Mises tests for multivariate independence. *The Annals of Statistics* **35**, 166–191.
- Kojadinovic, I. and Holmes, M. (2009), Tests of independence among continuous random vectors based on Cramér-von Mises functionals of the empirical copula process. *Journal of Multivariate Analysis* **100**, 1137–1154.

**See Also**

[indepTest](#), [serialIndepTest](#), [multSerialIndepTest](#), [dependogram](#).

**Examples**

```

## Consider the following example taken from
## Kojadinovic and Holmes (2008):

n <- 100

## Generate data
y <- matrix(rnorm(6*n),n,6)
y[,1] <- y[,2]/2 + sqrt(3)/2*y[,1]
y[,3] <- y[,4]/2 + sqrt(3)/2*y[,3]
y[,5] <- y[,6]/2 + sqrt(3)/2*y[,5]

nc <- normalCopula(0.3,dim=3)
x <- cbind(y,rCopula(n, nc),rCopula(n, nc))

x[,1] <- abs(x[,1]) * sign(x[,3] * x[,5])
x[,2] <- abs(x[,2]) * sign(x[,3] * x[,5])
x[,7] <- x[,7] + x[,10]
x[,8] <- x[,8] + x[,11]
x[,9] <- x[,9] + x[,12]

## Dimensions of the random vectors
d <- c(2,2,2,3,3)

## Run the test
test <- multIndepTest(x,d)
test

## Display the dependogram
dependogram(test,print=TRUE)

```

---

|                     |                                                                                   |
|---------------------|-----------------------------------------------------------------------------------|
| multSerialIndepTest | <i>Serial Independence Test for Multivariate Time Series via Empirical Copula</i> |
|---------------------|-----------------------------------------------------------------------------------|

---

**Description**

Analog of the serial independence test based on the empirical copula process proposed by Christian Genest and Bruno Rémillard (see [serialIndepTest](#)) for *multivariate* time series. The main difference comes from the fact that critical values and p-values are obtained through the bootstrap/permutation methodology, since, here, test statistics are not distribution-free.

**Usage**

```

multSerialIndepTest(x, lag.max, m = lag.max+1, N = 1000, alpha = 0.05,
  verbose = interactive())

```

**Arguments**

|         |                                                                                                                                                                                  |
|---------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x       | data frame or matrix of multivariate continuous time series whose serial independence is to be tested.                                                                           |
| lag.max | maximum lag.                                                                                                                                                                     |
| m       | maximum cardinality of the subsets of 'lags' for which a test statistic is to be computed. It makes sense to consider $m \ll \text{lag.max}+1$ especially when lag.max is large. |
| N       | number of bootstrap/permutation samples.                                                                                                                                         |
| alpha   | significance level used in the computation of the critical values for the test statistics.                                                                                       |
| verbose | a logical specifying if progress should be displayed via <a href="#">txtProgressBar</a> .                                                                                        |

**Details**

See the references below for more details, especially the last one.

The former argument `print.every` is deprecated and not supported anymore; use `verbose` instead.

**Value**

The function "multSerialIndepTest" returns an object of class "indepTest" whose attributes are: `subsets`, `statistics`, `critical.values`, `pvalues`, `fisher.pvalue` (a p-value resulting from a combination *à la* Fisher of the subset statistic p-values), `tippett.pvalue` (a p-value resulting from a combination *à la* Tippett of the subset statistic p-values), `alpha` (global significance level of the test), `beta` ( $1 - \text{beta}$  is the significance level per statistic), `global.statistic` (value of the global Cramér-von Mises statistic derived directly from the independence empirical copula process - see `In` in the last reference) and `global.statistic.pvalue` (corresponding p-value).

**References**

- Deheuvels, P. (1979) La fonction de dépendance empirique et ses propriétés: un test non paramétrique d'indépendance. *Acad. Roy. Belg. Bull. Cl. Sci.*, 5th Ser. **65**, 274–292.
- Deheuvels, P. (1981) A non parametric test for independence. *Publ. Inst. Statist. Univ. Paris* **26**, 29–50.
- Genest, C. and Rémillard, B. (2004) Tests of independence and randomness based on the empirical copula process. *Test* **13**, 335–369.
- Ghoubi, K., Kulperger, R., and Rémillard, B. (2001) A nonparametric test of serial independence for times series and residuals. *Journal of Multivariate Analysis* **79**, 191–218.
- Kojadinovic, I. and Yan, J. (2011) Tests of multivariate serial independence based on a Möbius decomposition of the independence empirical copula process. *Annals of the Institute of Statistical Mathematics* **63**, 347–373.

**See Also**

[serialIndepTest](#), [indepTest](#), [multIndepTest](#), [dependogram](#)



**Examples**

```
## A multivariate time series {minimal example for demo purposes}
d <- 2
n <- 100 # sample size *and* "burn-in" size
param <- 0.25
A <- matrix(param,d,d) # the bivariate AR(1)-matrix
set.seed(17)
ar <- matrix(rnorm(2*n * d), 2*n,d) # used as innovations
for (i in 2:(2*n))
  ar[i,] <- A %%% ar[i-1,] + ar[i,]
## drop burn-in :
x <- ar[(n+1):(2*n),]

## Run the test
test <- multSerialIndepTest(x,3)
test

## Display the dependogram
dependogram(test,print=TRUE)
```

---

Mvdc

*Multivariate Distributions Constructed from Copulas*


---

**Description**

Density, distribution function, and random generator for a multivariate distribution via copula and *parametric* margins.

For likelihood and fitting these distributions to data, see [fitMvdc](#).

**Usage**

```
mvdc(copula, margins, paramMargins, marginsIdentical = FALSE,
      check = TRUE, fixupNames = TRUE)
dMvdc(x, mvdc, log=FALSE)
pMvdc(x, mvdc)
rMvdc(n, mvdc)
```

**Arguments**

|                  |                                                                                                                                                                               |
|------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| copula           | an object of " <a href="#">copula</a> ".                                                                                                                                      |
| margins          | a <a href="#">character</a> vector specifying all the parametric marginal distributions. See details below.                                                                   |
| paramMargins     | a <a href="#">list</a> whose each component is a list (or numeric vectors) of named components, giving the parameter values of the marginal distributions. See details below. |
| marginsIdentical | logical variable restricting the marginal distributions to be identical.                                                                                                      |

|            |                                                                                                                                                                                                                      |
|------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| check      | logical indicating to apply quick checks about existence of margins “p*” and “d*” functions.                                                                                                                         |
| fixupNames | logical indicating if the parameters of the margins should get automatic names (from <code>formals(p&lt;mar_i&gt;)</code> ).                                                                                         |
| mvdc       | a “mvdc” object.                                                                                                                                                                                                     |
| x          | a numeric vector of length the copula dimension, say $d$ , or a matrix with the number of columns being $d$ , giving the coordinates of the points where the density or distribution function needs to be evaluated. |
| log        | logical indicating if the <code>log</code> density should be returned.                                                                                                                                               |
| n          | number of observations to be generated.                                                                                                                                                                              |

### Details

The characters in argument `margins` are used to construct density, distribution, and quantile function names. For example, `norm` can be used to specify marginal distribution, because `dnorm`, `pnorm`, and `qnorm` are all available.

A user-defined distribution, for example, `fancy`, can be used as margin *provided* that `dfancy`, `pfancy`, and `qfancy` are available.

Each component list in argument `paramMargins` is a `list` with named components which are used to specify the parameters of the marginal distributions. For example, the list

```
paramMargins = list(list(mean = 0, sd = 2), list(rate = 2))
```

can be used to specify that the first margin is normal with mean 0 and standard deviation 2, and the second margin is exponential with rate 2.

### Value

`mvdc()` constructs an object of class “mvdc”. `dMvdc()` gives the density, `pMvdc()` gives the cumulative distribution function, and `rMvdc()` generates random variates.

### Note

`mvdc()`, `fitMvdc`, etc, are only for *parametric* margins. If you do not want to model all margins parametrically, use the standard copula approach, transforming the data by their empirical margins via `pobs` and modelling the copula alone, e.g., using `fitCopula`, i.e., conceptually, using

```
fitCopula(.., pobs(x))
```

### See Also

`ellipCopula`, `archmCopula`; the classes `mvdc` and `copula`.

**Examples**

```
## construct a bivariate distribution whose marginals
## are normal and exponential respectively, coupled
## together via a normal copula
mv.NE <- mvdc(normalCopula(0.75), c("norm", "exp"),
              list(list(mean = 0, sd =2), list(rate = 2)))
dim(mv.NE)
mv.NE # using its print() / show() method

persp (mv.NE, dMvdc, xlim = c(-4, 4), ylim=c(0, 2), main = "dMvdc(mv.NE)")
persp (mv.NE, pMvdc, xlim = c(-4, 4), ylim=c(0, 2), main = "pMvdc(mv.NE)")
contour(mv.NE, dMvdc, xlim = c(-4, 4), ylim=c(0, 2))

# Generate (bivariate) random numbers from that, and visualize
x.samp <- rMvdc(250, mv.NE)
plot(x.samp)
summary(fx <- dMvdc(x.samp, mv.NE))
summary(Fx <- pMvdc(x.samp, mv.NE))
op <- par(mfcol=c(1,2))
pp <- persp(mv.NE, pMvdc, xlim = c(-5,5), ylim=c(0,2),
            main = "pMvdc(mv.NE)", ticktype="detail")
px <- copula::perspMvdc(x.samp, FUN = F.n, xlim = c(-5, 5), ylim = c(0, 2),
                       main = "F.n(x.samp)", ticktype="detail")
par(op)
all.equal(px, pp)# about 5% difference
```

mvdc-class

*Class "mvdc": Multivariate Distributions from Copulas***Description**

"mvdc" is a [class](#) representing **multivariate distributions** constructed via **copula** and margins, using Sklar's theorem.

**Objects from the Class**

Objects are typically created by [mvdc\(\)](#), or can be created by calls of the form `new("mvdc", ...)`.

**Slots**

**copula:** Object of class "[copula](#)", specifying the copula.

**margins:** Object of class "character", specifying the marginal distributions.

**paramMargins:** Object of class "list", whose each component is a list of named components, giving the parameter values of the marginal distributions. See [mvdc](#).

**marginsIdentical:** Object of class "logical", that, if TRUE, restricts the marginal distributions to be identical, default is FALSE.

**Methods**

**contour** signature(x = "mvdc"): ...

**dim** signature(x = "mvdc"): the dimension of the distribution; this is the same as dim(x@copula).

**persp** signature(x = "mvdc"): ...

**show** signature(object = "mvdc"): quite compactly display the content of the "mvdc" object.

**See Also**

[mvdc](#), also for examples; for fitting, [fitMvdc](#).

---

nacFrail.time

*Timing for Sampling Frailties of Nested Archimedean Copulas*

---

**Description**

This function provides measurements of user run times for the frailty variables involved in a nested Archimedean copula.

**Usage**

```
nacFrail.time(n, family, taus, digits = 3, verbose = FALSE)
```

**Arguments**

|         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |
|---------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| n       | integer specifying the sample size to be used for the random variates $V_0$ and $V_{01}$ .                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| family  | the Archimedean family (class " <a href="#">acopula</a> ") for which $V_0$ and $V_{01}$ are sampled.                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| taus    | <a href="#">numeric</a> vector of Kendall's taus. This vector is converted to a vector of copula parameters $\theta$ , which then serve as $\theta_0$ and $\theta_1$ for a three-dimensional fully nested Archimedean copula of the specified family. First, for each $\theta_0$ , n random variates $V_0$ are generated. Then, given the particular $\theta_0$ and the realizations $V_0$ , n random variates $V_{01}$ are generated for each $\theta_1$ fulfilling the sufficient nesting condition; see <a href="#">paraConstr</a> in <a href="#">acopula</a> . |
| digits  | number of digits for the output.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
| verbose | logical indicating if <code>nacFrail.time</code> output should generated while the random variates are generated (defaults to FALSE).                                                                                                                                                                                                                                                                                                                                                                                                                              |

**Value**

A  $k \times k$  matrix of user run time measurements in milliseconds ( $1000 * \text{system.time}(\cdot)[1]$ ) where  $k$  is `length(taus)`. The first column contains the run times for generating the  $V_0$ s. For the submatrix that remains if the first column is removed, row  $i$  (for  $\theta_{0i}$ ) contains the run times for the  $V_{01}$ s for a particular  $\theta_0$  and all the admissible  $\theta_1$ s.

**See Also**

The class [acopula](#) and our predefined "acopula" family objects in [acopula-families](#). For some timings on a standard notebook, see [demo](#)(timings) (or the file 'timings.R' in the demo folder).

**Examples**

```
## takes about 7 seconds:% so we rather test a much smaller set in R CMD check

nacFrail.time(10000, "Gumbel", taus= c(0.05,(1:9)/10, 0.95))

system.time(
print( nacFrail.time(1000, "Gumbel", taus = c(0.5,1,6,9)/10) )
)
```

---

|                |                                                       |
|----------------|-------------------------------------------------------|
| nacopula-class | <i>Class "nacopula" of Nested Archimedean Copulas</i> |
|----------------|-------------------------------------------------------|

---

**Description**

Class of nested Archimedean Copulas, "nacopula", and its *specification* "outer\_nacopula" differ only by the validation method, which is stricter for the outer(most) copula (the root copula).

**Objects from the Class**

Objects can be created by calls of the form `new("nacopula", ...)`, which is only intended for experts. Root copulas are typically constructed by [onacopula](#)(.).

**Slots**

**copula**: an object of class "[acopula](#)", denoting the top-level Archimedean copula of the nested Archimedean copula, that is, the root copula.

**comp**: an [integer](#) vector (possibly of length 0) of indices of components in 1:d which are not nested Archimedean copulas. Here, *d* denotes the *dimension* of the random vectors under consideration; see the `dim()` method below.

**childCops**: a (possibly empty) [list](#) of further nested Archimedean copulas (child copulas), that is, objects of class "nacopula". The "nacopula" objects therefore contain "[acopula](#)" objects as special cases.

**Methods**

**dim** signature(`x = "nacopula"`): returns the dimension *d* of the random vector *U* following *x*.

**show** signature("nacopula"): calling [printNacopula](#) for a compact overview of the nested Archimedean copula under consideration.

**See Also**

[onacopula](#) for building (outer) "nacopula" objects. For the class definition of the copula component, see [acopula](#).

**Examples**

```
## nacopula and outer_nacopula class information
showClass("nacopula")
showClass("outer_nacopula")

## Construct a three-dimensional nested Frank copula with parameters
## chosen such that the Kendall's tau of the respective bivariate margins
## are 0.2 and 0.5.
theta0 <- copFrank@iTau(.2)
theta1 <- copFrank@iTau(.5)
C3 <- onacopula("F", C(theta0, 1, C(theta1, c(2,3))))

C3 # displaying it, using show(C3); see help(printNacopula)

## What is the dimension of this copula?
dim(C3)

## What are the indices of direct components of the root copula?
C3@comp

## How does the list of child nested Archimedean copulas look like?
C3@childCops # only one child for this copula, components 2, 3
```

---

nacPairthetas

*Pairwise Thetas of Nested Archimedean Copulas*


---

**Description**

Return a  $d \times d$  matrix of pairwise thetas for a nested Archimedean copula ([nacopula](#)) of dimension  $d$ .

**Usage**

```
nacPairthetas(x)
```

**Arguments**

`x` an (outer) nacopula (with thetas sets).

**Value**

a  $(d \times d)$  matrix of thetas, say  $T$ , where  $T[j,k] = \text{theta of the bivariate Archimedean copula } C(U_j, U_k)$ .

**See Also**

the class [nacopula](#) (with its `dim` method).

**Examples**

```

## test with
options(width=97)

(mm <- rnacModel("Gumbel", d=15, pr.comp = 0.25, order="random"))
stopifnot(isSymmetric(PT <- nacPairthetas(mm)))
round(PT, 2)

## The tau's -- "Kendall's correlation matrix" :
round(copGumbel@tau(PT), 2)

## do this several times:
m1 <- rnacModel("Gumbel", d=15, pr.comp = 1/8, order="seq")
stopifnot(isSymmetric(PT <- nacPairthetas(m1)))
m1; PT

m100 <- rnacModel("Gumbel", d= 100, pr.comp = 1/16, order="seq")
system.time(PT <- nacPairthetas(m100))# how slow {non-optimal algorithm}?
##-- very fast, still!
stopifnot(isSymmetric(PT))
m100

## image(PT)# not ok -- want one color per theta
nt <- length(th0 <- unique(sort(PT[!is.na(PT)])))
th1 <- c(th0[1]/2, th0, 1.25*th0[nt])
ths <- (th1[-1]+th1[-(nt+2)])/2
image(log(PT), breaks = ths, col = heat.colors(nt))

## Nicer and easier:
require(Matrix)
image(as(log(PT),"Matrix"), main = "log( nacPairthetas( m100 ))",
      useAbs=FALSE, useRaster=TRUE, border=NA)

```

---

nesdepth

*Nesting Depth of a Nested Archimedean Copula ("nacopula")*


---

**Description**

Compute the nesting depth of a nested Archimedean copula which is the length of the longest branch in the tree representation of the copula, and hence at least one.

**Usage**

```
nesdepth(x)
```

**Arguments**

x                    object of class "[nacopula](#)".

**Value**

an integer, the nesting depth of the nested Archimedean copula. An (unnested) Archimedean copula has depth 1.

**See Also**

[dim](#) of nacopulas.

**Examples**

```
F2 <- onacopula("F", C(1.9, 1, C(4.5, c(2,3))))
F2
F3 <- onacopula("Clayton", C(1.5, 3:1,
                             C(2.5, 4:5,
                               C(15, 9:6))))
nesdepth(F2) # 2
nesdepth(F3) # 3
```

---

onacopula

*Constructing (Outer) Nested Archimedean Copulas*


---

**Description**

Constructing (outer) nested Archimedean copulas (class [outer\\_nacopula](#)) is most conveniently done via `onacopula()`, using a nested  $C(\dots)$  notation.

Slightly less conveniently, but with the option to pass a [list](#) structure, `onacopulaL()` can be used, typically from inside another function programmatically.

**Usage**

```
onacopula (family, nacStructure)
onacopulaL(family, nacList)
nac2list(x)
```

**Arguments**

**family** either a [character](#) string, the short or longer form of the Archimedean family name (for example, "Clayton" or simply "C"); see the [acopula-families](#) documentation, or an [acopula](#) family object.

**nacStructure** a "formula" of the form

$$C(\theta, c(i_1, \dots, i_c), \text{list}(C(\dots), \dots, C(\dots))).$$

Note that  $C()$  has (maximally) three arguments: the first is the copula parameter (vector)  $\theta$ , the second a (possibly empty) vector of integer indices of components (for the comp slot in [nacopulas](#)), and finally a (possibly empty) list of child copulas, each specified with in the  $C(\dots)$  notation themselves.



naclist            a **list** of length 3 (or 2), with elements

1. theta:  $\theta$
2. comp: components  $c(i_1, \dots, i_c)$
3. children: a **list** which must be a naclist itself and may be missing to denote the empty list().

x                    an "nacopula", (typically "outer\_nacopula") object.

### Value

onacopula[L](): An outer nested Archimedean copula object, that is, of class "outer\_nacopula".  
 nac2list: a **list** exactly like the naclist argument to onacopulaL.

### References

Those of the Archimedean families, for example, [copGumbel](#).

### See Also

The class definitions "nacopula", "outer\_nacopula", and "acopula".

### Examples

```
## Construct a ten-dimensional Joe copula with parameter such that
## Kendall's tau equals 0.5
theta <- copJoe@iTau(0.5)
C10 <- onacopula("J",C(theta,1:10))

## Equivalent construction with onacopulaL():
C10. <- onacopulaL("J",list(theta,1:10))
stopifnot(identical(C10, C10.),
           identical(nac2list(C10), list(theta, 1:10)))

## Construct a three-dimensional nested Gumbel copula with parameters
## such that Kendall's tau of the respective bivariate margins are 0.2
## and 0.5.
theta0 <- copGumbel@iTau(.2)
theta1 <- copGumbel@iTau(.5)
C3 <- onacopula("G", C(theta0, 1, C(theta1, c(2,3))))

## Equivalent construction with onacopulaL():
str(NAlis <- list(theta0, 1, list(list(theta1, c(2,3))))))
C3. <- onacopulaL("Gumbel", NAlis)
stopifnot(identical(C3, C3.))

## An exercise: assume you got the copula specs as character string:
na3spec <- "C(theta0, 1, C(theta1, c(2,3)))"
na3call <- str2lang(na3spec) # == parse(text = na3spec)[[1]]
C3.s <- onacopula("Gumbel", na3call)
stopifnot(identical(C3, C3.s))

## Good error message if the component ("coordinate") indices are wrong
```

```

## or do not match:
err <- try(onacopula("G", C(theta0, 2, C(theta1, c(3,2)))))

## Compute the probability of falling in [0,.01]^3 for this copula
pCopula(rep(.01,3), C3)

## Compute the probability of falling in the cube [.99,1]^3
prob(C3, rep(.99, 3), rep(1, 3))

## Construct a 6-dimensional, partially nested Gumbel copula of the form
## C_0(C_1(u_1, u_2), C_2(u_3, u_4), C_3(u_5, u_6))
theta <- 2:5
copG <- onacopulaL("Gumbel", list(theta[1], NULL, list(list(theta[2], c(1,2)),
                                                    list(theta[3], c(3,4)),
                                                    list(theta[4], c(5,6)))))

set.seed(1)
U <- rCopula(5000, copG)
pairs(U, pch=".", gap=0, labels = as.expression( lapply(1:dim(copG),
                                                    function(j) bquote(italic(U[(j)]))) ))

```

---

opower

*Outer Power Transformation of Archimedean Copulas*


---

### Description

Build a new Archimedean copula by applying the outer power transformation to a given Archimedean copula.

### Usage

```
opower(copbase, thetabase)
```

### Arguments

|           |                                                                                                                                                                           |
|-----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| copbase   | a "base" copula, that is, a copula of class <a href="#">acopula</a> . Must be one of the predefined families.                                                             |
| thetabase | the univariate parameter $\theta$ for the generator of the base copula copbase. Hence, the copula which is transformed is fixed, that is, does not depend on a parameter. |

### Value

a new [acopula](#) object, namely the outer power copula based on the provided copula family copbase with fixed parameter thetabase. The transform introduces a parameter theta, so one obtains a parametric Archimedean family object as return value.

The [environment](#) of all function slots contains objects cOP (which is the outer power copula itself), copbase, and thetabase.

## References

Hofert, M. (2010), *Sampling Nested Archimedean Copulas with Applications to CDO Pricing*, Suedwestdeutscher Verlag fuer Hochschulschriften AG & Co. KG.

## See Also

The class [acopula](#) and our predefined "acopula" family objects in [acopula-families](#).

## Examples

```
## Construct an outer power Clayton copula with parameter thetabase such
## that Kendall's tau equals 0.2
thetabase <- copClayton@iTau(0.2)
opC <- opower(copClayton, thetabase) # "acopula" obj. (unspecified theta)

## Construct a 3d nested Archimedean copula based on opC, that is, a nested
## outer power Clayton copula. The parameters theta are chosen such that
## Kendall's tau equals 0.4 and 0.6 for the outer and inner sector,
## respectively.
theta0 <- opC@iTau(0.4)
theta1 <- opC@iTau(0.6)
opC3d <- onacopulaL(opC, list(theta0, 1, list(list(theta1, 2:3))))
## or opC3d <- onacopula(opC, C(theta0, 1, C(theta1, c(2,3))))

## Compute the corresponding lower and upper tail-dependence coefficients
rbind(theta0 = c(
  lambdaL = opC@lambdaL(theta0),
  lambdaU = opC@lambdaU(theta0) # => opC3d has upper tail dependence
),
  theta1 = c(
  lambdaL = opC@lambdaL(theta1),
  lambdaU = opC@lambdaU(theta1) # => opC3d has upper tail dependence
))

## Sample opC3d
n <- 1000
U <- rnacopula(n, opC3d)

## Plot the generated vectors of random variates of the nested outer
## power Clayton copula.
splom2(U)

## Construct such random variates "by hand"
## (1) draw V0 and V01
V0 <- opC@V0(n, theta0)
V01 <- opC@V01(V0, theta0, theta1)
## (2) build U
U <- cbind(
  opC@psi(rexp(n)/V0, theta0),
  opC@psi(rexp(n)/V01, theta1),
  opC@psi(rexp(n)/V01, theta1))
```

---

|        |                                                                                  |
|--------|----------------------------------------------------------------------------------|
| pairs2 | <i>Scatter-Plot Matrix ('pairs') for Copula Distributions with Nice Defaults</i> |
|--------|----------------------------------------------------------------------------------|

---

### Description

A version of **graphics'** package `pairs()`, particularly useful for visualizing dependence in multivariate (copula) data.

### Usage

```
pairs2(x, labels = NULL, labels.null.lab = "U", ...)
```

### Arguments

|                              |                                                                                                                                                                                                |
|------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>x</code>               | a numeric <code>matrix</code> or an R object for which <code>as.matrix(x)</code> returns such a matrix.                                                                                        |
| <code>labels</code>          | the variable names, typically unspecified.                                                                                                                                                     |
| <code>labels.null.lab</code> | the <code>character</code> string determining the “base name” of the variable labels in case <code>labels</code> is <code>NULL</code> and <code>x</code> does not have all column names given. |
| <code>...</code>             | further arguments, passed to <code>pairs()</code> .                                                                                                                                            |

### Value

```
invisible()
```

### See Also

`splom2()` for a similar function based on `splom()`.

### Examples

```
## Create a 100 x 7 matrix of random variates from a t distribution
## with four degrees of freedom and plot the generated data
U <- matrix(rt(700, df = 4), ncol = 7)
pairs2(U, pch = ".")
```

---

pairsRosenblatt      *Plots for Graphical GOF Test via Pairwise Rosenblatt Transforms*

---

### Description

pairsCollist() creates a **list** containing information about colors for a given matrix of (approximate aka “pseudo”) p-values. These colors are used in pairsRosenblatt() for visualizing a graphical goodness-of-fit test based on pairwise Rosenblatt transformed data.

### Usage

```
pairsRosenblatt(cu.u, pvalueMat=pviTest(pairwiseIndepTest(cu.u)),
  method = c("scatter", "QQchisq", "QQgamma",
    "PPchisq", "PPgamma", "none"),
  g1, g2, col = "B&W.contrast",
  collist = pairsCollist(pvalueMat, col=col),
  main=NULL,
  sub = gpviString(pvalueMat, name = "pp-values"),
  panel = NULL, do.qqline = TRUE,
  keyOpt = list(title="pp-value", rug.at=pvalueMat), ...)
```

```
pairsCollist(P, pdiv = c(1e-04, 0.001, 0.01, 0.05, 0.1, 0.5),
  signif.P = 0.05, pmin0 = 1e-05, bucketCols = NULL,
  fgColMat = NULL, bgColMat = NULL, col = "B&W.contrast",
  BWcutoff = 170,
  bg.col = c("ETHCL", "zurich", "zurich.by.fog", "baby",
    "heat", "greenish"),
  bg.ncol.gap = floor(length(pdiv)/3),
  bg.col.bottom = NULL, bg.col.top = NULL, ...)
```

### Arguments

|           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
|-----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| cu.u      | $(n, d, d)$ -array of pairwise Rosenblatt-transformed observations as returned by <code>pairwiseCcop()</code> .                                                                                                                                                                                                                                                                                                                                                                                                   |
| pvalueMat | $(d, d)$ -matrix of p-values (or pp-values).                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| method    | <b>character</b> indicating the plot method to be used. Currently possible are:<br>"scatter" a simple scatter plot.<br>"QQchisq" a Q-Q plot after a map to the $\chi^2$ distribution.<br>"QQgamma" a Q-Q plot after a map to the gamma distribution.<br>"PPchisq" a P-P plot after a map to the $\chi^2$ distribution.<br>"PPgamma" a P-P plot after a map to the gamma distribution.<br>"none" no points are plotted.<br>Note: These methods merely just set g1 and g2 correctly; see the code for more details. |
| g1        | <b>function</b> from $[0, 1]^n \rightarrow [0, 1]^n$ applied to "x" for plotting in one panel.                                                                                                                                                                                                                                                                                                                                                                                                                    |

|               |                                                                                                                                                                                                                                                 |
|---------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| g2            | function from $[0, 1]^{n \times 2} \rightarrow [0, 1]^n$ applied to "y" for plotting in one panel.                                                                                                                                              |
| colList       | list of colors and information as returned by pairsColList().                                                                                                                                                                                   |
| main          | title.                                                                                                                                                                                                                                          |
| sub           | sub-title with a smart default containing a global (p)p-value.                                                                                                                                                                                  |
| panel         | a panel function as for pairs, or, by default, NULL, where the panel is set as points or "points + qqline" if the method is "QQ..." and do.qqline is true.                                                                                      |
| do.qqline     | if method = "QQ...", specify if the plot panels should also draw a qqline().                                                                                                                                                                    |
| keyOpt        | argument passed to .pairsCond() for options for the key.                                                                                                                                                                                        |
| ...           | additional arguments passed to .pairsCond() (for pairsRosenblatt()) and to heat_hcl() (for pairsColList; used to generate the color palette), see Details.                                                                                      |
| P             | $d \times d$ matrix of p-values.                                                                                                                                                                                                                |
| pdiv          | numeric vector of strictly increasing p-values in (0,1) that determine the "buckets" for the background colors of .pairsCond() which creates the pairs-like goodness-of-fit plot.                                                               |
| signif.P      | significance level (must be an element of pdiv).                                                                                                                                                                                                |
| pmin0         | a numeric indicating the lower endpoint of the p-value buckets if pmin is zero. If set to 0, the lowest value of the p-value buckets will also be 0.<br>Note that pmin0 should be in (0, min(pdiv)) when using pairsColList() for .pairsCond(). |
| bucketCols    | vector of length as pdiv containing the colors for the buckets. If not specified, either bg.col.bottom and bg.col.top are used (if provided) or bg.col (if provided).                                                                           |
| fgColMat      | ( $d, d$ )-matrix with foreground colors (the default will be black if the background color is bright and white if it is dark; see also BWcutoff).                                                                                              |
| bgColMat      | ( $d, d$ )-matrix of background colors; do not change this unless you know what you are doing.                                                                                                                                                  |
| col           | foreground color (defaults to "B&W.contrast" which switches black/white according to BWcutoff), passed to .pairsCond(). If colList is not specified, this color is used to construct the points' color.                                         |
| BWcutoff      | number in (0, 255) for switching foreground color if col="B&W.contrast".                                                                                                                                                                        |
| bg.col        | color scheme for the background colors.                                                                                                                                                                                                         |
| bg.ncol.gap   | number of colors left out as "gap" for color buckets below/above signif.P (to make significance/non-significance more visible).                                                                                                                 |
| bg.col.bottom | vector of length 3 containing a HCL color specification. If bg.col.bottom is provided and bucketCols is not, bg.col.bottom is used as the color for the bucket of smallest p-values.                                                            |
| bg.col.top    | vector of length 3 containing a HCL color specification. If bg.col.top is provided and bucketCols is not, bg.col.top is used as the color for the bucket of largest p-values.                                                                   |

## Details

Extra arguments of `pairsRosenblatt()` are passed to `.pairsCond()`, these notably may include `key`, `true` by default, which draws a color key for the colors used as panel background encoding (pseudo) p-values.

`pairsColList()` is basically an auxiliary function to specify the colors used in the graphical goodness-of-fit test as conducted by `pairsRosenblatt()`. The latter is described in detail in Hofert and Mächler (2013). See also `demo(gof_graph)`.

## Value

`pairsRosenblatt`: invisibly, the result of `.pairsCond()`.

`pairsColList`: a named `list` with components

`fgColMat` `matrix` of foreground colors.

`bgColMat` `matrix` of background colors (corresponding to P).

`bucketCols` `vector` containing the colors corresponding to `pvalueBuckets` as described above.

`pvalueBuckets` `vector` containing the endpoints of the p-value buckets.

## References

Hofert, M. and Mächler, M. (2014) A graphical goodness-of-fit test for dependence models in higher dimensions; *Journal of Computational and Graphical Statistics*, **23**(3), 700–716. doi:10.1080/10618600.2013.812518

## See Also

`pairwiseCcop()` for the tools behind the scenes. `demo(gof_graph)` for examples.

## Examples

```
## 2-dim example {d = 2} =====
##
## "t" Copula with 22. degrees of freedom; and (pairwise) tau = 0.5
nu <- 22 # degrees of freedom
## Define the multivariate distribution
tCop <- ellipCopula("t", param=iTau(ellipCopula("t", df=nu), tau = 0.5),
                  dim=2, df=nu)
set.seed(19)
X <- qexp(rCopula(n = 400, tCop))

## H0 (wrongly): a Normal copula, with correct tau
copH0 <- ellipCopula("normal", param=iTau(ellipCopula("normal"), tau = 0.5))

## create array of pairwise copH0-transformed data columns
cu.u <- pairwiseCcop(pobs(X), copula = copH0)

## compute pairwise matrix of p-values and corresponding colors
pwIT <- pairwiseIndepTest(cu.u, N=200) # (d,d)-matrix of test results
```

```

round(pmat <- pviTest(pwIT), 3) # pick out p-values
## .286 and .077
pairsRosenblatt(cu.u, pvalueMat= pmat)

### A shortened version of demo(gof_graph) -----

N <- 32 ## too small, for "testing"; realistically, use a larger one:
if(FALSE)
N <- 100

## 5d Gumbel copula #####

n <- 250 # sample size
d <- 5 # dimension
family <- "Gumbel" # copula family
tau <- 0.5
set.seed(17)
## define and sample the copula (= H0 copula), build pseudo-observations
cop <- getAcop(family)
th <- cop@iTau(tau) # correct parameter value
copH0 <- onacopulaL(family, list(th, 1:d)) # define H0 copula
U. <- pobs(rCopula(n, cop=copH0))

## create array of pairwise copH0-transformed data columns
cu.u <- pairwiseCcop(U., copula = copH0)

## compute pairwise matrix of p-values and corresponding colors
pwIT <- pairwiseIndepTest(cu.u, N=N, verbose=interactive()) # (d,d)-matrix of test results
round(pmat <- pviTest(pwIT), 3) # pick out p-values
## Here (with seed=1): no significant ones, smallest = 0.0603

## Plots -----

## plain (too large plot symbols here)
pairsRosenblatt(cu.u, pvalueMat=pmat, pch=".")

## with title, no subtitle
pwRoto <- "Pairwise Rosenblatt transformed observations"
pairsRosenblatt(cu.u, pvalueMat=pmat, pch=".", main=pwRoto, sub=NULL)

## two-line title including expressions, and centered
title <- list(paste(pwRoto, "to test"),
             substitute(italic(H[0]:C~~bold("is Gumbel with"~~tau==tau.)),
                list(tau.=tau)))
line.main <- c(4, 1.4)
pairsRosenblatt(cu.u, pvalueMat=pmat, pch=".",
               main=title, line.main=line.main, main.centered=TRUE)

## Q-Q plots -- can, in general, better detect outliers
pairsRosenblatt(cu.u, pvalueMat=pmat, method="QQchisq", cex=0.2)

```



**Description**

Methods for function `persp` to draw perspective plots (of two dimensional distributions from package `copula`).

**Usage**

```
## S4 method for signature 'Copula'
persp(x, FUN, n.grid = 26, delta = 0,
      xlab = "u1", ylab = "u2",
      zlab = deparse(substitute(FUN))[1], zlim = NULL,
      theta = -30, phi = 30, expand = 0.618,
      ticktype = "detail", ...)
## S4 method for signature 'mvdc'
persp(x, FUN, xlim, ylim, n.grid = 26,
      xlab = "x1", ylab = "x2", zlab = deparse(substitute(FUN))[1],
      theta = -30, phi = 30, expand = 0.618,
      ticktype = "detail", ...)
```

**Arguments**

|                                                                        |                                                                                                                                                                                                                                                   |
|------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>x</code>                                                         | a "Copula" or a "mvdc" object.                                                                                                                                                                                                                    |
| <code>FUN</code>                                                       | the function to be plotted; typically <code>dCopula</code> or <code>pCopula</code> .                                                                                                                                                              |
| <code>n.grid</code>                                                    | the number of grid points used in each dimension. This can be a vector of length two, giving the number of grid points used in x- and y-direction, respectively; the function <code>FUN</code> will be evaluated on the corresponding (x,y)-grid. |
| <code>delta</code>                                                     | A small number in $[0, \frac{1}{2}]$ influencing the evaluation boundaries. The x- and y-vectors will have the range $[0+\text{delta}, 1-\text{delta}]$ , the default being $[0, 1]$ .                                                            |
| <code>xlim, ylim</code>                                                | The range of the x and y variables, respectively.                                                                                                                                                                                                 |
| <code>xlab, ylab, zlab, zlim, theta, phi, expand, ticktype, ...</code> | Arguments for (the default method of) <code>persp()</code> , the ones enumerated here all with <i>different</i> defaults than there.                                                                                                              |

**Value**

`invisible`; a list with the following components:

|                    |                                                                                                       |
|--------------------|-------------------------------------------------------------------------------------------------------|
| <code>x, y</code>  | The numeric vectors, as passed to <code>persp.default</code> .                                        |
| <code>z</code>     | The matrix of evaluated <code>FUN</code> values on the grid as passed to <code>persp.default</code> . |
| <code>persp</code> | the $4 \times 4$ transformation matrix returned by <code>persp.default</code> .                       |

**Methods**

Perspective plots for both "copula" or "mvdc" objects, see x in the *Arguments* section.

**See Also**

The [contour-methods](#) for drawing contour lines of the same functions.

**Examples**

```
persp(claytonCopula(2), pCopula, main = "CDF of claytonCopula(2)")
persp( frankCopula(1.5), dCopula, main = "Density of frankCopula(1.5)")
persp( frankCopula(1.5), dCopula, main = "c_[frank(1.5)](.)", zlim = c(0,2))

## Examples with negative tau:
(th1 <- iTau(amhCopula(), -0.1))
persp(amhCopula(th1), dCopula)
persp(amhCopula(th1), pCopula, ticktype = "simple") # no axis ticks
persp( frankCopula(iTau( frankCopula(), -0.1)), dCopula)
persp(claytonCopula(iTau(claytonCopula(), -0.1)), dCopula)
##
cCop.2 <- function(u, copula, ...) cCopula(u, copula, ...)[,2]
persp( amhCopula(iTau( amhCopula(), -0.1)), cCop.2, main="cCop(AMH...)[,2]")
persp( frankCopula(iTau( frankCopula(), -0.1)), cCop.2, main="cCop(frankC)[,2]")
## and Clayton also looks "the same" ...

## MVDC Examples -----
mvNN <- mvdc(gumbelCopula(3), c("norm", "norm"),
            list(list(mean = 0, sd = 1), list(mean = 1)))
persp(mvNN, dMvdc, xlim=c(-2, 2), ylim=c(-1, 3), main = "Density")
persp(mvNN, pMvdc, xlim=c(-2, 2), ylim=c(-1, 3), main = "Cumulative Distr.")
```

---

plackettCopula

*Construction of a Plackett Copula*

---

**Description**

Constructs a Plackett copula (class "plackettCopula") with its corresponding parameter.

**Usage**

```
plackettCopula(param)
```

**Arguments**

param                    a number (numeric vector of length one) specifying the *non negative* parameter.

**Value**

A Plackett copula object of class "plackettCopula".

**References**

Plackett, R. L. (1965). A Class of Bivariate Distributions. *Journal of the American Statistical Association* **60**, 516–522.

**See Also**

The "[plackettCopula](#)" class; [ellipCopula](#), [archmCopula](#).

**Examples**

```
plackett.cop <- plackettCopula(param=2)
lambda(plackett.cop) # 0 0 : no tail dependencies
## For really large param values (here, 1e20 and Inf are equivalent):
set.seed(1); Xe20 <- rCopula(5000, plackettCopula(1e20))
set.seed(1); Xinf <- rCopula(5000, plackettCopula(Inf))
stopifnot(all.equal(Xe20, Xinf))
```

---

plackettCopula-class    *Class "plackettCopula" of Plackett Copulas*

---

**Description**

The Plackett copula class.

**Objects from the Class**

Objects can be created by `new("plackettCopula", ...)` but are typically produced by `plackettCopula(alpha)`.

**Slots**

**dimension:** Numeric (scalar), the dimension of the copula.  
**exprdist:** a length two [expression](#) with expressions for the CDF and PDF of the copula.  
**parameters:** a number (numeric vector of length one) specifying the *non negative* parameter.  
**param.names:** the "[character](#)" string "alpha".  
**param.lowbnd:** the number 0.  
**param.upbnd:** the number Inf.  
**fullname:** (deprecated; do not use!)

**Methods**

Typical copula methods work, see "[plackettCopula](#)" and use `methods(class = "plackettCopula")`.

**Extends**

Class "plackettCopula" extends class "[copula](#)" directly.

**References**

Nelsen, R. B. (2006), *An introduction to Copulas*, Springer, New York.

**See Also**

[copula-class](#), [plackettCopula](#).

**Examples**

```
str(plackettCopula())

plackettCopula()@exprdist[["cdf"]]
methods(class = "plackettCopula")
contourplot2(plackettCopula(7), pCopula)
wireframe2(plackettCopula(5), dCopula, main= "plackettCopula(5)")
```

---

plot-methods

*Methods for 'plot' in Package 'copula'*

---

**Description**

Methods for [plot\(\)](#) to draw a scatter plot of a random sample from bivariate distributions from package **copula**.

**Usage**

```
## S4 method for signature 'Copula,ANY'
plot(x, n, xlim = 0:1, ylim = 0:1,
      xlab = quote(U[1]), ylab = quote(U[2]), main = NULL, ...)
## S4 method for signature 'mvdc,ANY'
plot(x, n, xlim = NULL, ylim = NULL,
      xlab = quote(X[1]), ylab = quote(X[2]), ...)
```

**Arguments**

|                         |                                                                                                                                                                                                                                                                           |
|-------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>x</code>          | a <i>bivariate "matrix"</i> , " <a href="#">data.frame</a> ", " <a href="#">Copula</a> " or a " <a href="#">mvdc</a> " object.                                                                                                                                            |
| <code>n</code>          | when <code>x</code> is not matrix-like: The sample size of the random sample drawn from <code>x</code> .                                                                                                                                                                  |
| <code>xlim, ylim</code> | the x- and y-axis limits.                                                                                                                                                                                                                                                 |
| <code>xlab, ylab</code> | the x- and y-axis labels.                                                                                                                                                                                                                                                 |
| <code>main</code>       | the main title; when true, shows the call's <code>x</code> "expression". By default, when NULL and the <code>x</code> expression matches " <a href="#">[Cc]opula</a> " that expression is used as well. This smart default is somewhat experimental; feedback is welcome. |
| <code>...</code>        | additional arguments passed to <a href="#">plot</a> methods, i.e., typically <a href="#">plot.default</a> .                                                                                                                                                               |

**Value**

invisible().

**See Also**

`splom2()` for a scatter-plot *matrix* based on `splom()`.

**Examples**

```
## For 2-dim. 'copula' objects -----
## Plot uses n copula samples
n <- 1000 # sample size
set.seed(271) # (reproducibility)
plot(tCopula(-0.8, df = 1.25), n = n) # automatic main title!

nu <- 3 # degrees of freedom
tau <- 0.5 # Kendall's tau
th <- iTau(tCopula(df = nu), tau) # corresponding parameter
cop <- tCopula(th, df = nu) # define 2-d copula object
plot(cop, n = n)

## For 2-dim. 'mvdc' objects -----
mvNN <- mvdc(cop, c("norm", "norm"),
             list(list(mean = 0, sd = 1), list(mean = 1)))
plot(mvNN, n = n)
```

---

pnacopula

*Evaluation of (Nested) Archimedean Copulas*

---

**Description**

For a (nested) Archimedean copula (object of class `nacopula`) `x`, `pCopula(u, x)` (or also currently still `pnacopula(x, u)`) evaluates the copula `x` at the given vector or matrix `u`.

**Usage**

```
## S4 method for signature 'matrix,nacopula'
pCopula(u, copula, ...)

## *Deprecated*:
pnacopula(x, u)
```

**Arguments**

|                        |                                                                                                                                                 |
|------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>copula, x</code> | (nested) Archimedean copula of dimension $d$ , that is, an object of class <code>nacopula</code> , typically from <code>onacopula(...)</code> . |
| <code>u</code>         | a <code>numeric</code> vector of length $d$ or matrix with $d$ columns.                                                                         |
| <code>...</code>       | unused: potential optional arguments passed from and to methods.                                                                                |

**Details**

The value of an Archimedean copula  $C$  with generator  $\psi$  at  $u$  is given by

$$C(\mathbf{u}) = \psi(\psi^{-1}(u_1) + \cdots + \psi^{-1}(u_d)), \mathbf{u} \in [0, 1]^d.$$

The value of a nested Archimedean copula is defined similarly. Note that a  $d$ -dimensional copula is called *nested Archimedean* if it is an Archimedean copula with arguments possibly replaced by other nested Archimedean copulas.

**Value**

A `numeric` in  $[0, 1]$  which is the copula evaluated at  $u$ . (Currently not parallelized.)

**Note**

`pCopula(u, copula)` is a *generic* function with methods for *all* our copula classes, see `pCopula`.

**Examples**

```
## Construct a three-dimensional nested Joe copula with parameters
## chosen such that the Kendall's tau of the respective bivariate margins
## are 0.2 and 0.5.
theta0 <- copJoe@iTau(.2)
theta1 <- copJoe@iTau(.5)
C3 <- onacopula("J", C(theta0, 1, C(theta1, c(2,3))))

## Evaluate this copula at the vector u
u <- c(.7,.8,.6)
pCopula(u, C3)

## Evaluate this copula at the matrix v
v <- matrix(runif(300), ncol=3)
pCopula(v, C3)

## Back-compatibility check
stopifnot(identical( pCopula (u, C3), suppressWarnings(
  pnacopula(C3, u))),
  identical( pCopula (v, C3), suppressWarnings(
    pnacopula(C3, v))))
```

---

pobs

*Pseudo-Observations*

---

**Description**

Compute the pseudo-observations for the given data matrix.

**Usage**

```
pobs(x, na.last = "keep",
     ties.method = eval(formals(rank)$ties.method), lower.tail = TRUE)
```

**Arguments**

|                          |                                                                                                                                                                      |
|--------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>x</code>           | $n \times d$ -matrix (or $d$ -vector) of random variates to be converted to pseudo-observations.                                                                     |
| <code>na.last</code>     | string passed to <code>rank</code> ; see there.                                                                                                                      |
| <code>ties.method</code> | <b>character</b> string specifying how ranks should be computed if there are ties in any of the coordinate samples of <code>x</code> ; passed to <code>rank</code> . |
| <code>lower.tail</code>  | <b>logical</b> which, if <code>FALSE</code> , returns the pseudo-observations when applying the empirical marginal survival functions.                               |

**Details**

Given  $n$  realizations  $\mathbf{x}_i = (x_{i1}, \dots, x_{id})^T$ ,  $i \in \{1, \dots, n\}$  of a random vector  $\mathbf{X}$ , the pseudo-observations are defined via  $u_{ij} = r_{ij}/(n+1)$  for  $i \in \{1, \dots, n\}$  and  $j \in \{1, \dots, d\}$ , where  $r_{ij}$  denotes the rank of  $x_{ij}$  among all  $x_{kj}$ ,  $k \in \{1, \dots, n\}$ . When there are no ties in any of the coordinate samples of `x`, the pseudo-observations can thus also be computed by component-wise applying the marginal empirical distribution functions to the data and scaling the result by  $n/(n+1)$ . This asymptotically negligible scaling factor is used to force the variates to fall inside the open unit hypercube, for example, to avoid problems with density evaluation at the boundaries. Note that `pobs(, lower.tail=FALSE)` simply returns `1-pobs()`.

**Value**

**matrix** (or **vector**) of the same dimensions as `x` containing the pseudo-observations.

**Examples**

```
## Simple definition of the function:
pobs
## Draw from a multivariate normal distribution
d <- 10
set.seed(1)
P <- Matrix::nearPD(matrix(pmin(pmax(runif(d*d), 0.3), 0.99), ncol=d))$mat
diag(P) <- rep(1, d)
n <- 500
x <- MASS::mvrnorm(n, mu = rep(0, d), Sigma = P)

## Compute pseudo-observations (should roughly follow a Gauss
## copula with correlation matrix P)
u <- pobs(x)
plot(u[,5], u[,10], xlab=quote(italic(U)[1]), ylab=quote(italic(U)[2]))

## All components: pairwise plot
pairs(u, gap=0, pch=".", labels =
      as.expression( lapply(1:d, function(j) bquote(italic(U)[.](j)))) )
```

**Description**

Compute the polylogarithm function  $\text{Li}_s(z)$ , initially defined as the power series,

$$\text{Li}_s(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^s},$$

for  $|z| < 1$ , and then more generally (by analytic continuation) as

$$\text{Li}_1(z) = -\log(1 - z),$$

and

$$\text{Li}_{s+1}(z) = \int_0^z \frac{\text{Li}_s(t)}{t} dt.$$

Currently, mainly the case of negative integer  $s$  is well supported, as that is used for some of the Archimedean copula densities.

For  $s = 2$ ,  $\text{Li}_2(z)$  is also called ‘dilogarithm’ or “Spence’s function”. The “default” method uses the `dilog` or `complex_dilog` function from package `gsl`, respectively when  $s = 2$ .

Also compute the Debye\_ $n$  functions, for  $n = 1$  and  $n = 2$ , in a slightly more general manner than the `gsl` package functions `debye_1` and `debye_2` (which cannot deal with non-finite  $x$ .)

**Usage**

```
polylog(z, s,
        method = c("default", "sum", "negI-s-Stirling",
                  "negI-s-Eulerian", "negI-s-asympt-w"),
        logarithm = FALSE, is.log.z = FALSE, is.logmlog = FALSE,
        asymp.w.order = 0, n.sum)

debye1(x)
debye2(x)
```

**Arguments**

|                         |                                                                                                                                                                                           |
|-------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>z</code>          | numeric or complex vector                                                                                                                                                                 |
| <code>s</code>          | complex number; current implementation is aimed at $s \in \{0, -1, \dots\}$                                                                                                               |
| <code>method</code>     | a string specifying the algorithm to be used.                                                                                                                                             |
| <code>logarithm</code>  | logical specified to return $\log(\text{Li}(\cdot))$ instead of $\text{Li}(\cdot)$                                                                                                        |
| <code>is.log.z</code>   | logical; if TRUE, the specified <code>z</code> argument is really $w = \log(z)$ ; that is, we compute $\text{Li}_s(\exp(w))$ , and we typically have $w < 0$ , or equivalently, $z < 1$ . |
| <code>is.logmlog</code> | logical; if TRUE, the specified argument <code>z</code> is $lw = \log(-w) = \log(-\log(z))$ (where as above, $w = \log(z)$ ).                                                             |



asympt.w.order    currently only default is implemented.  
 n.sum             for method="sum" only: the number of terms used.  
 x                  numeric vector, may contain Inf, NA, and negative values.

### Details

Almost entirely taken from <https://en.wikipedia.org/wiki/Polylogarithm>:

For integer values of the polylogarithm order, the following explicit expressions are obtained by repeated application of  $z \frac{\partial}{\partial z}$  to  $\text{Li}_1(z)$ :

$$\text{Li}_1(z) = -\log(1-z), \quad \text{Li}_0(z) = \frac{z}{1-z}, \quad \text{Li}_{-1}(z) = \frac{z}{(1-z)^2}, \quad \text{Li}_{-2}(z) = \frac{z(1+z)}{(1-z)^3},$$

$$\text{Li}_{-3}(z) = \frac{z(1+4z+z^2)}{(1-z)^4}, \text{ etc.}$$

Accordingly, the polylogarithm reduces to a ratio of polynomials in  $z$ , and is therefore a rational function of  $z$ , for all nonpositive integer orders. The general case may be expressed as a finite sum:

$$\text{Li}_{-n}(z) = \left( z \frac{\partial}{\partial z} \right)^n \frac{z}{1-z} = \sum_{k=0}^n k! S(n+1, k+1) \left( \frac{z}{1-z} \right)^{k+1} \quad (n = 0, 1, 2, \dots),$$

where  $S(n, k)$  are the Stirling numbers of the second kind.

Equivalent formulae applicable to negative integer orders are (Wood 1992, § 6) ...

$$\text{Li}_{-n}(z) = \frac{1}{(1-z)^{n+1}} \sum_{k=0}^{n-1} \langle n \rangle_k z^{n-k} = \frac{z \sum_{k=0}^{n-1} \langle n \rangle_k z^k}{(1-z)^{n+1}}, \quad (n = 1, 2, 3, \dots),$$

where  $\langle n \rangle_k$  are the Eulerian numbers; see also [Eulerian](#).

### Value

numeric/complex vector as  $z$ , or  $x$ , respectively.

### References

- Wikipedia (2011) *Polylogarithm*, <https://en.wikipedia.org/wiki/Polylogarithm>.
- Wood, D. C. (June 1992). The Computation of Polylogarithms. Technical Report 15-92. Canterbury, UK: University of Kent Computing Laboratory. <https://www.cs.kent.ac.uk/pubs/1992/110/>.
- Apostol, T. M. (2010), "*Polylogarithm*", in the NIST Handbook of Mathematical Functions, <https://dlmf.nist.gov/25.12>
- Lewin, L. (1981). *Polylogarithms and Associated Functions*. New York: North-Holland. ISBN 0-444-00550-1.
- For Debye functions: Levin (1981) above, and  
 Wikipedia (2014) *Debye function*, [https://en.wikipedia.org/wiki/Debye\\_function](https://en.wikipedia.org/wiki/Debye_function).

**See Also**

The polylogarithm is used in MLE for some Archimedean copulas; see [emle](#);

The Debye functions are used for [tau](#) or [rho](#) computations of the Frank copula.

**Examples**

```
## The dilogarithm, polylog(z, s = 2) = Li_2(.) -- mathmatically defined on C \ [1, Inf)
## so x -> 1 is a limit case:
polylog(z = 1, s = 2)
## in the limit, should be equal to
pi^2 / 6

## Default method uses GSL's dilog():
rLi2 <- curve(polylog(x, 2), -5, 1, n= 1+ 6*64, col=2, lwd=2)
abline(c(0,1), h=0,v=0:1, lty=3, col="gray40")
## "sum" method gives the same for |z| < 1 and large number of terms:
ii <- which(abs(rLi2$x) < 1)
stopifnot(all.equal(rLi2$y[ii],
                    polylog(rLi2$x[ii], 2, "sum", n.sum = 1e5),
                    tolerance = 1e-15))

z1 <- c(0.95, 0.99, 0.995, 0.999, 0.9999)
L <- polylog(z1, s=-3,method="negI-s-Euler") # close to Inf
LL <- polylog(log(z1), s=-3,method="negI-s-Euler",is.log.z=TRUE)
LLL <- polylog(log(-log(z1)),s=-3,method="negI-s-Euler",is.logmlog=TRUE)
all.equal(L, LL)
all.equal(L, LLL)

p.Li <- function(s.set, from = -2.6, to = 1/4, ylim = c(-1, 0.5),
                colors = c("orange","brown", palette()), n = 201, ...)
{
  s.set <- sort(s.set, decreasing = TRUE)
  s <- s.set[1] # <_ for auto-ylab
  curve(polylog(x, s, method="negI-s-Stirling"), from, to,
        col=colors[1], ylim=ylim, n=n, ...)
  abline(h=0,v=0, col="gray")
  for(is in seq_along(s.set)[-1])
    curve(polylog(x, s=s.set[is], method="negI-s-Stirling"),
          add=TRUE, col = colors[is], n=n)
  s <- rev(s.set)
  legend("bottomright",paste("s =",s), col=colors[2-s], lty=1, bty="n")
}

## yellow is unbearable (on white):
palette(local({p <- palette(); p[p=="yellow"] <- "goldenrod"; p}))

## Wikipedia page plot (+/-):
p.Li(1:-3, ylim= c(-.8, 0.6), colors = c(2:4,6:7))

## and a bit more:
p.Li(1:-5)
```

```
## For the range we need it:
ccol <- c(NA,NA, rep(palette(),10))
p.Li(-1:-20, from=0, to=.99, colors=ccol, ylim = c(0, 10))
## log-y scale:
p.Li(-1:-20, from=0, to=.99, colors=ccol, ylim = c(.01, 1e7),
     log = "y", yaxt = "n")
if(require(sfsmisc)) eaxis(2) else axis(2)
```

---

 polynEval

*Evaluate Polynomials*


---

### Description

Evaluate a univariate polynomial at  $x$  (typically a vector), that is, compute, for a given vector of coefficients `coef`, the polynomial  $\text{coef}[1] + \text{coef}[2]*x + \dots + \text{coef}[p+1]*x^p$ .

### Usage

```
polynEval(coef, x)
```

### Arguments

|                   |                                                                               |
|-------------------|-------------------------------------------------------------------------------|
| <code>coef</code> | numeric vector. If a vector, $x$ can be an array and the result matches $x$ . |
| <code>x</code>    | numeric vector or array.                                                      |

### Details

The stable Horner rule is used for evaluation.

Using the C code speeds up the already fast R code available in `polyn.eval()` in package **sfsmisc**.

### Value

numeric vector or array, with the same dimensions as  $x$ , containing the polynomial values  $p(x)$ .

### See Also

For a much more sophisticated treatment of polynomials, use the `polynom` package (for example, evaluation can be done via `predict.polynomial`).

### Examples

```
polynEval(c(1,-2,1), x = -2:7) # (x - 1)^2
polynEval(c(0, 24, -50, 35, -10, 1),
          x = matrix(0:5, 2,3)) # 5 zeros!
```

---

|               |                                                                           |
|---------------|---------------------------------------------------------------------------|
| printNacopula | <i>Print Compact Overview of a Nested Archimedean Copula ("nacopula")</i> |
|---------------|---------------------------------------------------------------------------|

---

### Description

Print a compact overview of a nested Archimedean copula, that is, an object of class "[nacopula](#)". Calling `printNacopula` explicitly allows to customize the printing behavior. Otherwise, the `show()` method calls `printNacopula` with default arguments only.

### Usage

```
printNacopula(x, labelKids=NA, deltaInd=, indent.str="",
             digits=getOption("digits"),
             width=getOption("width"), ...)
```

### Arguments

|                            |                                                                                                                                                                                                                                      |
|----------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>x</code>             | an R object of class <a href="#">nacopula</a> .                                                                                                                                                                                      |
| <code>labelKids</code>     | logical specifying if child copulas should be labeled; If NA (as per default), on each level, children are labeled only if they are not only-child.                                                                                  |
| <code>deltaInd</code>      | by how much should each child be indented <i>more</i> than its parent? (non-negative integer). The default is three with <code>labelKids</code> being the default or TRUE, otherwise it is five (for <code>labelKids=FALSE</code> ). |
| <code>indent.str</code>    | a <a href="#">character</a> string specifying the indentation, that is, the string that should be <i>prepended</i> on the first line of output, and determine the amount of blanks for the remaining lines.                          |
| <code>digits, width</code> | number of significant digits, and desired print width; see <a href="#">print.default</a> .                                                                                                                                           |
| <code>...</code>           | potentially further arguments, passed to methods.                                                                                                                                                                                    |

### Value

invisibly, `x`.

### Examples

```
C8 <- onacopula("F", C(1.9, 1,
                    list(K1 = C(5.7, c(2,5)),
                        abc= C(5.0, c(3,4,6)),
                        list(L2 = C(11.5, 7:8))))))
C8 # -> printNacopula(C8)
printNacopula(C8, delta=10)
printNacopula(C8, labelKids=TRUE)
```

**Description**

Compute probabilities of a  $d$ -dimensional random vector  $U$  distributed according to a given copula  $x$  to fall in a hypercube  $(l, u]$ , where  $l$  and  $u$  denote the lower and upper corners of the hypercube, respectively.

**Usage**

```
prob(x, l, u)
```

**Arguments**

$x$  copula of dimension  $d$ , that is, an object inheriting from [Copula](#).  
 $l, u$   $d$ -dimensional, **numeric**, lower and upper hypercube boundaries, respectively, satisfying  $0 \leq l_i \leq u_i \leq 1$ , for  $i \in 1, \dots, d$ .

**Value**

A **numeric** in  $[0, 1]$  which is the probability  $P(l_i < U_i \leq u_i)$ .

**See Also**

[pCopula\(.\)](#).

**Examples**

```
## Construct a three-dimensional nested Joe copula with parameters
## chosen such that the Kendall's tau of the respective bivariate margins
## are 0.2 and 0.5.
theta0 <- copJoe@iTau(.2)
theta1 <- copJoe@iTau(.5)
C3 <- onacopula("J", C(theta0, 1, C(theta1, c(2,3))))

## Compute the probability of a random vector distributed according to
## this copula to fall inside the cube with lower point l and upper
## point u.
l <- c(.7,.8,.6)
u <- c(1,1,1)
prob(C3, l, u)

## ditto for a bivariate normal copula with rho = 0.8 :
prob(normalCopula(0.8), c(.2,.4), c(.3,.6))
```

**Description**

A Q-Q plot (possibly) with rugs and pointwise approximate (via the Central Limit Theorem) two-sided  $1-\alpha$  confidence intervals.

**Usage**

```
qqplot2(x, qF, log = "", qqline.args = if(log=="x" || log=="y")
  list(untf=TRUE) else list(),
  rug.args = list(tcl=-0.6*par("tcl")),
  alpha = 0.05, CI.args = list(col="gray40"),
  CI.mtext = list(text=paste0("Pointwise asymptotic ", 100*(1-alpha),
    "% confidence intervals"), side=4,
    cex=0.6*par("cex.main"), adj=0, col="gray40"),
  main = quote(bold(italic(F)~"Q-Q plot")),
  main.args = list(text=main, side=3, line=1.1, cex=par("cex.main"),
    font=par("font.main"), adj=par("adj"), xpd=NA),
  xlab = "Theoretical quantiles", ylab = "Sample quantiles",
  file="", width=6, height=6, ...)
```

**Arguments**

|             |                                                                                                                                                                            |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x           | numeric.                                                                                                                                                                   |
| qF          | (theoretical) quantile function against which the Q-Q plot is created.                                                                                                     |
| log         | character string indicating whether log-scale should be used; see <code>?plot.default</code> .                                                                             |
| qqline.args | argument <code>list</code> passed to <code>qqline()</code> for creating the Q-Q line. Use <code>qqline.args=NULL</code> to omit the Q-Q line.                              |
| rug.args    | argument <code>list</code> passed to <code>rug()</code> for creating the rugs. Use <code>rug.args=NULL</code> to omit the rugs.                                            |
| alpha       | significance level.                                                                                                                                                        |
| CI.args     | argument <code>list</code> passed to <code>lines()</code> for plotting the confidence intervals. Use <code>CI.args=NULL</code> to omit the confidence intervals.           |
| CI.mtext    | argument <code>list</code> passed to <code>mtext()</code> for plotting information about the confidence intervals. Use <code>CI.mtext=NULL</code> to omit the information. |
| main        | title (can be an expression; use "" for no title).                                                                                                                         |
| main.args   | argument <code>list</code> passed to <code>mtext()</code> for plotting the title. Use <code>main.args=NULL</code> to omit the title.                                       |
| xlab        | x axis label.                                                                                                                                                              |
| ylab        | y axis label.                                                                                                                                                              |
| file        | file name including the extension “.pdf”.                                                                                                                                  |

width            width parameter of `pdf()`.  
 height          height parameter of `pdf()`.  
 ...             additional arguments passed to `plot()` based for plotting the points.

### Details

See the source code for how the confidence intervals are constructed precisely.

### Value

`invisible()`.

### See Also

`plot()` for the underlying plot function, `qqline()` for how the Q-Q line is implemented, `rug()` for how the rugs are constructed, `lines()` for how the confidence intervals are drawn, and `mtext()` for how the title and information about the confidence intervals is printed. `pdf()` for plotting to pdf.

### Examples

```
n <- 250
df <- 7
set.seed(1)
x <- rchisq(n, df=df)

## Q-Q plot against the true quantiles (of a chi^2_3 distribution)
qqplot2(x, qF = function(p) qchisq(p, df=df),
        main = substitute(bold(italic(chi[NU])~~"Q-Q Plot"), list(NU=df)))

## in log-log scale
qqplot2(x, qF = function(p) qchisq(p, df=df), log="xy",
        main = substitute(bold(italic(chi[NU])~~"Q-Q Plot"), list(NU=df)))

## Q-Q plot against wrong quantiles (of an Exp(1) distribution)
qqplot2(x, qF=qexp, main = quote(bold(Exp(1)~~"Q-Q Plot")))
```

---

radSymTest

*Test of Exchangeability for a Bivariate Copula*

---

### Description

Test for assessing the radial symmetry of the underlying multivariate copula based on the empirical copula. The test statistic is a multivariate extension of the definition adopted in the first reference. An approximate p-value for the test statistic is obtained by means of a appropriate *bootstrap* which can take the presence of ties in the component series of the data into account; see the second reference.

**Usage**

```
radSymTest(x, N = 1000, ties = NA)
```

**Arguments**

|                   |                                                                                                                                                                                                                                          |
|-------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>x</code>    | a data matrix that will be transformed to pseudo-observations.                                                                                                                                                                           |
| <code>N</code>    | number of bootstrap iterations to be used to simulate realizations of the test statistic under the null hypothesis.                                                                                                                      |
| <code>ties</code> | logical; if TRUE, the bootstrap procedure is adapted to the presence of ties in any of the coordinate samples of <code>x</code> ; the default value of NA indicates that the presence/absence of ties will be checked for automatically. |

**Details**

More details are available in the second reference.

**Value**

An object of `class` `hctest` which is a list, some of the components of which are

|                        |                                    |
|------------------------|------------------------------------|
| <code>statistic</code> | value of the test statistic.       |
| <code>p.value</code>   | corresponding approximate p-value. |

**References**

Genest, C. and G. Nešlehová, J. (2014). On tests of radial symmetry for bivariate copulas. *Statistical Papers* **55**, 1107–1119.

Kojadinovic, I. (2017). Some copula inference procedures adapted to the presence of ties. *Computational Statistics and Data Analysis* **112**, 24–41, <https://arxiv.org/abs/1609.05519>.

**See Also**

[exchTest](#), [exchEVTTest](#), [gofCopula](#).

**Examples**

```
## Data from radially symmetric copulas
radSymTest(rCopula(200, frankCopula(3)))
radSymTest(rCopula(200, normalCopula(0.7, dim = 3)))

## Data from non radially symmetric copulas
radSymTest(rCopula(200, claytonCopula(3)))
radSymTest(rCopula(200, gumbelCopula(2, dim=3)))
```



rdj

*Daily Returns of Three Stocks in the Dow Jones***Description**

Five years of daily log-returns (from 1996 to 2000) of Intel (INTC), Microsoft (MSFT) and General Electric (GE) stocks. These data were analysed in Chapter 5 of McNeil, Frey and Embrechts (2005).

**Usage**

```
data(rdj, package="copula")
```

**Format**

A data frame of 1262 daily log-returns from 1996 to 2000.

Date the date, of class "Date".

INTC daily log-return of the Intel stock

MSFT daily log-return of the Microsoft stock

GE daily log-return of the General Electric

**References**

McNeil, A. J., Frey, R., and Embrechts, P. (2005). *Quantitative Risk Management: Concepts, Techniques, Tools*. Princeton University Press.

**Examples**

```
data(rdj)
str(rdj)# 'Date' is of class "Date"

with(rdj, {
  matplot(Date, rdj[,-1], type = "o", xaxt = "n", ylim = .15* c(-1,1),
    main = paste("rdj - data; n =", nrow(rdj)))
  Axis(Date, side=1)
})
legend("top", paste(1:3, names(rdj[,-1])), col=1:3, lty=1:3, bty="n")

x <- rdj[, -1] # '-1' : not the Date
## a t-copula (with a vague initial guess of Rho entries)
tCop <- tCopula(rep(.2, 3), dim=3, dispstr="un", df=10, df.fixed=TRUE)
ft <- fitCopula(tCop, data = pobs(x))
ft
ft@copula # the fitted t-copula as tCopula object
system.time(
g.C <- gofCopula(claytonCopula(dim=3), as.matrix(x), simulation = "mult")
) ## 5.3 sec
```

```
system.time(
g.t <- gofCopula(ft@copula, as.matrix(x), simulation = "mult")
) ## 8.1 sec
```

retstable

*Sampling Exponentially Tilted Stable Distributions***Description**

Generating random variates of an exponentially tilted stable distribution of the form

$$\tilde{S}(\alpha, 1, (\cos(\alpha\pi/2)V_0)^{1/\alpha}, V_0\mathbf{1}_{\{\alpha=1\}}, h\mathbf{1}_{\{\alpha\neq 1\}}; 1),$$

with parameters  $\alpha \in (0, 1]$ ,  $V_0 \in (0, \infty)$ , and  $h \in [0, \infty)$  and corresponding Laplace-Stieltjes transform

$$\exp(-V_0((h+t)^\alpha - h^\alpha)), t \in [0, \infty);$$

see the references for more details about this distribution.

**Usage**

```
retstable(alpha, V0, h = 1, method = NULL)
retstableR(alpha, V0, h = 1)
```

**Arguments**

|        |                                                                                                                                                                                                                                                                   |
|--------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| alpha  | parameter in $(0, 1]$ .                                                                                                                                                                                                                                           |
| V0     | vector of values in $(0, \infty)$ (for example, when sampling nested Clayton copulas, these are random variates from $F_0$ ), that is, the distribution corresponding to $\psi_0$ .                                                                               |
| h      | parameter in $[0, \infty)$ .                                                                                                                                                                                                                                      |
| method | a character string denoting the method to use, currently either "MH" (Marius Hofert's algorithm) or "LD" (Luc Devroye's algorithm). By default, when NULL, a smart choice is made to use the fastest of these methods depending on the specific values of $V_0$ . |

**Details**

retstableR is a pure R version of "MH", however, not as fast as retstable (implemented in C, based on both methods) and therefore not recommended in simulations when run time matters.

**Value**

A vector of variates from  $\tilde{S}(\alpha, 1, \dots)$ ; see above.

## References

- Devroye, L. (2009) Random variate generation for exponentially and polynomially tilted stable distributions, *ACM Transactions on Modeling and Computer Simulation* **19**, 18, 1–20.
- Hofert, M. (2011) Efficiently sampling nested Archimedean copulas, *Computational Statistics & Data Analysis* **55**, 57–70.
- Hofert, M. (2012), Sampling exponentially tilted stable distributions, *ACM Transactions on Modeling and Computer Simulation* **22**, 1.

## See Also

[rstable1](#) for sampling stable distributions.

## Examples

```
## Draw random variates from an exponentially tilted stable distribution
## with given alpha, V0, and h = 1
alpha <- .2
V0 <- rgamma(200, 1)
rETS <- retstable(alpha, V0)

## Distribution plot the random variates -- log-scaled
hist(log(rETS), prob=TRUE)
lines(density(log(rETS)), col=2)
rug (log(rETS))
```

---

rF01FrankJoe

*Sample Univariate Distributions Involved in Nested Frank and Joe Copulas*

---

## Description

rF01Frank: Generate a vector of random variates  $V_{01} \sim F_{01}$  with Laplace-Stieltjes transform

$$\psi_{01}(t; V_0) = \left( \frac{1 - (1 - \exp(-t)(1 - e^{-\theta_1}))^{\theta_0/\theta_1}}{1 - e^{-\theta_0}} \right)^{V_0}.$$

for the given realizations  $V_0$  of Frank's  $F_0$  and the parameters  $\theta_0, \theta_1 \in (0, \infty)$  such that  $\theta_0 \leq \theta_1$ . This distribution appears on sampling nested Frank copulas. The parameter `rej` is used to determine the cut-off point of two algorithms that are involved in sampling  $F_{01}$ . If `rej` <  $V_0\theta_0(1 - e^{-\theta_0})^{V_0-1}$  a rejection from  $F_{01}$  of Joe is applied (see rF01Joe; the meaning of the parameter `approx` is explained below), otherwise a sum is sampled with a logarithmic envelope for each summand.

rF01Joe: Generate a vector of random variates  $V_{01} \sim F_{01}$  with Laplace-Stieltjes transform

$$\psi_{01}(t; V_0) = (1 - (1 - \exp(-t))^\alpha)^{V_0}.$$

for the given realizations  $V_0$  of Joe's  $F_0$  and the parameter  $\alpha \in (0, 1]$ . This distribution appears on sampling nested Joe copulas. Here,  $\alpha = \theta_0/\theta_1$ , where  $\theta_0, \theta_1 \in [1, \infty)$  such that  $\theta_0 \leq \theta_1$ . The

parameter `approx` denotes the largest number of summands in the sum-representation of  $V_{01}$  before the asymptotic

$$V_{01} = V_0^{1/\alpha} S(\alpha, 1, \cos^{1/\alpha}(\alpha\pi/2), \mathbf{1}_{\{\alpha=1\}}; 1)$$

is used to sample  $V_{01}$ .

### Usage

```
rF01Frank(V0, theta0, theta1, rej, approx)
rF01Joe(V0, alpha, approx)
```

### Arguments

`V0` a vector of random variates from  $F_0$ .  
`theta0, theta1, alpha` parameters  $\theta_0, \theta_1$  and  $\alpha$  as described above.  
`rej` parameter value as described above.  
`approx` parameter value as described above.

### Value

A vector of positive [integers](#) of length `n` containing the generated random variates.

### References

Hofert, M. (2011). Efficiently sampling nested Archimedean copulas. *Computational Statistics & Data Analysis* **55**, 57–70.

### See Also

[rFFrank](#), [rFJoe](#), [rSibuya](#), and [rnacopula](#).  
[rnacopula](#)

### Examples

```
## Sample n random variates V0 ~ F0 for Frank and Joe with parameter
## chosen such that Kendall's tau equals 0.2 and plot histogram
n <- 1000
theta0.F <- copFrank@iTau(0.2)
V0.F <- copFrank@V0(n, theta0.F)
hist(log(V0.F), prob=TRUE); lines(density(log(V0.F)), col=2, lwd=2)
theta0.J <- copJoe@iTau(0.2)
V0.J <- copJoe@V0(n, theta0.J)
hist(log(V0.J), prob=TRUE); lines(density(log(V0.J)), col=2, lwd=2)

## Sample corresponding V01 ~ F01 for Frank and Joe and plot histogram
## copFrank@V01 calls rF01Frank(V0, theta0, theta1, rej=1, approx=10000)
## copJoe@V01 calls rF01Joe(V0, alpha, approx=10000)
theta1.F <- copFrank@iTau(0.5)
V01.F <- copFrank@V01(V0.F, theta0.F, theta1.F)
```

```
hist(log(V01.F), prob=TRUE); lines(density(log(V01.F)), col=2, lwd=2)
theta1.J <- copJoe@iTau(0.5)
V01.J <- copJoe@V01(V0.J, theta0.J, theta1.J)
hist(log(V01.J), prob=TRUE); lines(density(log(V01.J)), col=2, lwd=2)
```

rFFrankJoe

*Sampling Distribution F for Frank and Joe***Description**

Generate a vector of variates  $V \sim F$  from the distribution function  $F$  with Laplace-Stieltjes transform

$$(1 - (1 - \exp(-t)(1 - e^{-\theta_1}))^\alpha) / (1 - e^{-\theta_0}),$$

for Frank, or

$$1 - (1 - \exp(-t))^\alpha,$$

for Joe, respectively, where  $\theta_0$  and  $\theta_1$  denote two parameters of Frank (that is,  $\theta_0, \theta_1 \in (0, \infty)$ ) and Joe (that is,  $\theta_0, \theta_1 \in [1, \infty)$ ) satisfying  $\theta_0 \leq \theta_1$  and  $\alpha = \theta_0/\theta_1$ .

**Usage**

```
rFFrank(n, theta0, theta1, rej)
rFJoe(n, alpha)
```

**Arguments**

|        |                                                                                                                                                        |
|--------|--------------------------------------------------------------------------------------------------------------------------------------------------------|
| n      | number of variates from $F$ .                                                                                                                          |
| theta0 | parameter $\theta_0$ .                                                                                                                                 |
| theta1 | parameter $\theta_1$ .                                                                                                                                 |
| rej    | method switch for rFFrank: if theta0 > rej a rejection from Joe's family (Sibuya distribution) is applied (otherwise, a logarithmic envelope is used). |
| alpha  | parameter $\alpha = \theta_0/\theta_1$ in $(0, 1]$ for rFJoe.                                                                                          |

**Details**

rFFrank(n, theta0, theta1, rej) calls [rF01Frank](#)(rep(1,n), theta0, theta1, rej, 1) and rFJoe(n, alpha) calls [rSibuya](#)(n, alpha).

**Value**

numeric vector of random variates  $V$  of length n.

**See Also**

[rF01Frank](#), [rF01Joe](#), also for references. [rSibuya](#), and [rncopula](#).

**Examples**

```
## Simple definition of the functions:
rFFrank
rFJoe
```

---

rlog

*Sampling Logarithmic Distributions*


---

**Description**

Generating random variates from a Log(p) distribution with probability mass function

$$p_k = \frac{p^k}{-\log(1-p)^k}, \quad k \in \mathbf{N},$$

where  $p \in (0, 1)$ . The implemented algorithm is the one named “LK” in Kemp (1981).

**Usage**

```
rlog(n, p, Ip = 1 - p)
```

**Arguments**

|    |                                                                          |
|----|--------------------------------------------------------------------------|
| n  | sample size, that is, length of the resulting vector of random variates. |
| p  | parameter in (0, 1).                                                     |
| Ip | = 1 - p, possibly more accurate, e.g, when $p \approx 1$ .               |

**Details**

For documentation and didactical purposes, rlogR is a pure-R implementation of rlog. However, rlogR is not as fast as rlog (the latter being implemented in C).

**Value**

A vector of positive [integers](#) of length n containing the generated random variates.

**Note**

In the **copula** package, the Log(p) distribution is needed only for generating Frank copula observations, namely in `copFrank@V0()`, where  $p = 1 - \exp(-\theta)$ , i.e.,  $p = -\expm1(-\theta)$  and  $Ip = \exp(-\theta)$ .

For large  $\theta$  it would be desirable to pass  $-\theta$  to `rlog()` instead of p. This has not yet been implemented.

**References**

Kemp, A. W. (1981), Efficient Generation of Logarithmically Distributed Pseudo-Random Variables, *Journal of the Royal Statistical Society: Series C (Applied Statistics)* **30**, 3, 249–253.

**Examples**

```

## Sample n random variates from a Log(p) distribution and plot a
## "histogram"
n <- 1000
p <- .5
X <- rlog(n, p)
table(X) ## distribution on the integers {1, 2, ..}
## ==> The following plot is more reasonable than a hist(X, prob=TRUE) :
plot(table(X)/n, type="h", lwd=10, col="gray70")

## case closer to numerical boundary:
lV <- log10(V <- rlog(10000, Ip = 2e-9))# Ip = exp(-theta) <==> theta ~ 20
hV <- hist(lV, plot=FALSE)
dV <- density(lV)
## Plot density and histogram on log scale with nice axis labeling & ticks:
plot(dV, xaxt="n", ylim = c(0, max(hV$density, dV$y)),
     main = "Density of [log-transformed] Log(p), p=0.999999..")
abline(h=0, lty=3); rug(lV); lines(hV, freq=FALSE, col = "light blue"); lines(dV)
rx <- range(pretty(par("usr")[1:2]))
sx <- outer(1:9, 10^(max(0,rx[1]):rx[2]))
axis(1, at=log10(sx), labels= FALSE, tcl = -0.3)
axis(1, at=log10(sx[1,]), labels= formatC(sx[1,]), tcl = -0.75)

```

rnacModel

*Random nacopula Model***Description**

Randomly construct a nested Archimedean copula model,

**Usage**

```
rnacModel(family, d, pr.comp, rtau0 = function() rbeta(1, 2,4),
          order=c("random", "each", "seq"), digits.theta = 2)
```

**Arguments**

|              |                                                                                                      |
|--------------|------------------------------------------------------------------------------------------------------|
| family       | the Archimedean family                                                                               |
| d            | integer $\geq 2$ ; the dimension                                                                     |
| pr.comp      | probability of a direct component on each level                                                      |
| rtau0        | a <a href="#">function</a> to generate a (random) tau, corresponding to theta0, the outermost theta. |
| order        | string indicating how the component IDs are selected.                                                |
| digits.theta | integer specifying the number of digits to round the theta values.                                   |

**Value**

an object of [outer\\_nacopula](#).

**See Also**

[rnacopula](#) for generating  $d$ -dimensional observations from an (outer) [nacopula](#), e.g., from the *result* of `rnacModel()`.

**Examples**

```
## Implicitly tests the function {with validity of outer_nacopula ..}
set.seed(11)
for(i in 1:40) {
  m1 <- rnacModel("Gumbel", d=sample(20:25, 1), pr.comp = 0.3,
    rtau0 = function() 0.25)
  m2 <- rnacModel("Joe", d=3, pr.comp = 0.1, order="each")
  mC <- rnacModel("Clayton", d=20, pr.comp = 0.3,
    rtau0 = function() runif(1, 0.1, 0.5))
  mF <- rnacModel("Frank", d=sample(20:25, 1), pr.comp = 0.3, order="seq")
}
```

---

 rnacopula

*Sampling Nested Archimedean Copulas*


---

**Description**

Random number generation for nested Archimedean copulas (of class [outer\\_nacopula](#), specifically), aka *sampling* nested Archimedean copulas will generate  $n$  random vectors of dimension  $d$  ( $= \dim(x)$ ).

**Usage**

```
rnacopula(n, copula, x, ...)
```

**Arguments**

|                     |                                                                                                                                |
|---------------------|--------------------------------------------------------------------------------------------------------------------------------|
| <code>n</code>      | integer specifying the sample size, that is, the number of copula-distributed random vectors $\mathbf{U}_i$ , to be generated. |
| <code>copula</code> | an R object of class " <a href="#">outer_nacopula</a> ", typically from <code>onacopula()</code> .                             |
| <code>x</code>      | only for back compatibility: former name of <code>copula</code> argument.                                                      |
| <code>...</code>    | possibly further arguments for the given copula family.                                                                        |

**Details**

The generation happens by calling `rnchild()` on each child copula (which itself recursively descends the tree implied by the nested Archimedean structure). The algorithm is based on a mixture representation of the generic distribution functions  $F_0$  and  $F_{01}$  and is presented in McNeil (2008) and Hofert (2011a). Details about how to efficiently sample the distribution functions  $F_0$  and  $F_{01}$  can be found in Hofert (2010), Hofert (2012), and Hofert and Mächler (2011).



**Value**

numeric matrix containing the generated vectors of random variates from the nested Archimedean copula object `copula`.

**References**

- McNeil, A. J. (2008). Sampling nested Archimedean copulas. *Journal of Statistical Computation and Simulation* **78**, 6, 567–581.
- Hofert, M. (2010). Efficiently sampling nested Archimedean copulas. *Computational Statistics & Data Analysis* **55**, 57–70.
- Hofert, M. (2012), A stochastic representation and sampling algorithm for nested Archimedean copulas. *Journal of Statistical Computation and Simulation*, **82**, 9, 1239–1255.
- Hofert, M. (2012). Sampling exponentially tilted stable distributions. *ACM Transactions on Modeling and Computer Simulation* **22**, 1 (3rd article).
- Hofert, M. and Mächler, M. (2011). Nested Archimedean Copulas Meet R: The `nacopula` Package. *Journal of Statistical Software* **39**, 9, 1–20.

**See Also**

`rnchild`; classes "`nacopula`" and "`outer_nacopula`"; see also `onacopula()`. `rnacModel` creates random `nacopula` models, i.e., the input copula for `rnacopula(n, copula)`.

Further, those of the Archimedean families, for example, `copGumbel`.

**Examples**

```
## Construct a three-dimensional nested Clayton copula with parameters
## chosen such that the Kendall's tau of the respective bivariate margins
## are 0.2 and 0.5 :
C3 <- onacopula("C", C(copClayton@iTau(0.2), 1,
                      C(copClayton@iTau(0.5), c(2,3))))
C3

## Sample n vectors of random variates from this copula. This involves
## sampling exponentially tilted stable distributions
n <- 1000
U <- rnacopula(n, C3)

## Plot the drawn vectors of random variates
splom2(U)
```

---

 rnchild

*Sampling Child 'nacopula's*


---

**Description**

Method for generating vectors of random numbers of nested Archimedean copulas which are child copulas.

**Usage**

```
rnchild(x, theta0, V0, ...)
```

**Arguments**

**x** an "nacopula" object, typically emerging from an "outer\_nacopula" object constructed with `onacopula()`.

**theta0** the parameter (vector) of the parent Archimedean copula which contains x as a child.

**V0** a **numeric** vector of realizations of  $V_0$  following  $F_0$  whose length determines the number of generated vectors, that is, for each realization  $V_0$ , a vector of variates from x is generated.

**...** possibly further arguments for the given copula family.

**Details**

The generation is done recursively, descending the tree implied by the nested Archimedean structure. The algorithm is based on a mixture representation and requires sampling  $V_{01} \sim F_{01}$  given random variates  $V_0 \sim F_0$ . Calling "rnchild" is only intended for experts. The typical call of this function takes place through `rnacopula()`.

**Value**

a list with components

**U** a **numeric** matrix containing the vector of random variates from the child copula. The number of rows of this matrix therefore equals the length of  $V_0$  and the number of columns corresponds to the dimension of the child copula.

**indcol** an **integer** vector of indices of U (the vector following a nested Archimedean copula of which x is a child) whose corresponding components of U are arguments of the nested Archimedean copula x.

**See Also**

`rnacopula`, also for the references. Further, classes "nacopula" and "outer\_nacopula"; see also `onacopula()`.

**Examples**

```
## Construct a three-dimensional nested Clayton copula with parameters
## chosen such that the Kendall's tau of the respective bivariate margins
## are 0.2 and 0.5.
theta0 <- copClayton@iTau(.2)
theta1 <- copClayton@iTau(.5)
C3 <- onacopula("C", C(theta0, 1, C(theta1, c(2,3))))
## Sample n random variates  $V_0 \sim F_0$  (a  $\text{Gamma}(1/\text{theta0}, 1)$  distribution)
n <- 1000
V0 <- copClayton@V0(n, theta0)
```

```
## Given these variates V0, sample the child copula, that is, the bivariate
## nested Clayton copula with parameter theta1
U23 <- rnchild(C3@childCops[[1]], theta0, V0)

## Now build the three-dimensional vectors of random variates by hand
U1 <- copClayton@psi(rexp(n)/V0, theta0)
U <- cbind(U1, U23$U)

## Plot the vectors of random variates from the three-dimensional nested
## Clayton copula
splom2(U)
```

rotCopula

*Construction and Class of Rotated aka Reflected Copulas***Description**

Constructs a “reflected” or “rotated” copula from an initial copula and a vector of logicals indicating which dimension to “flip”.

**Usage**

```
rotCopula(copula, flip = TRUE)
```

**Arguments**

**copula** an object of class “*Copula*”.

**flip** **logical** vector (of length 1 or `dim(copula)`) indicating which dimension should be “flipped”; by default, all the components are flipped, implying that the resulting copula is the “*survival copula*”.

**Value**

A “rotated” or “reflected” copula object of class “*rotCopula*”.

**Slots**

of a “*rotCopula*” object

**copula:** Object of class “*copula*”.

**flip:** **logical** vector of length  $d$  (the copula dimension) specifying which margins are flipped; corresponds to the `flip` argument of `rotCopula()`.

**dimension:** the copula dimension  $d$ , an **integer**.

**parameters:** **numeric** vector specifying the parameters.

**param.lowbnd, and param.upbnd:** numeric vector of the same length as `parameters`, specifying (component wise) bounds for each of the parameters.

**param.names:** **character** vector (of same length as `parameters`) with parameter names.

**fullname:** **deprecated**; a character string describing the rotated copula.

**Note**

When there are an even number of flips, the resulting copula can be seen as a *rotated* version of copula. In the other cases, e.g., `flip = c(FALSE, TRUE)` in 2d, it is rather a *reflected* or “mirrored” copula.

**See Also**

[fitCopula](#) for fitting such copulas to data, [gofCopula](#) for goodness-of-fit tests for such copulas.

**Examples**

```
## Two-dimensional examples: First the Clayton(3) survival copula:
survC <- rotCopula(claytonCopula(3)) # default: flip = 'all TRUE'
contourplot2(survC, dCopula)

## Now, a reflected Clayton copula:
r10C <- rotCopula(claytonCopula(3), flip = c(TRUE, FALSE))

contourplot2(r10C, dCopula, nlevels = 20, main = "dCopula(<rotCopula>")
contourplot2(r10C, pCopula, nlevels = 20, main = "pCopula(<rotCopula>")
rho(r10C)
tau(r10C) # -0.6

n <- 1000
u <- rCopula(n, r10C)
rho.n <- cor(u[,1], u[,2], method = "spearman")
tau.n <- cor(u[,1], u[,2], method = "kendall")

## "Calibration"
rc. <- rotCopula(claytonCopula(), flip = c(TRUE, FALSE))
iRho(rc., rho.n)
iTau(rc., tau.n)

## Fitting
fitCopula(rc., data = pobs(u), method = "irho")
fitCopula(rc., data = pobs(u), method = "itau")
fitCopula(rc., data = pobs(u), method = "mpl")

## Goodness-of-fit testing -- the first, parametric bootstrap, is *really* slow
## Not run: gofCopula(rc., x = u)
gofCopula(rc., x = u, simulation = "mult")

## A four-dimensional example: a rotated Frank copula
rf <- rotCopula(frankCopula(10, dim = 4),
               flip = c(TRUE, FALSE, TRUE, FALSE))

n <- 1000
u <- rCopula(n, rf)
splom2(u)

pCopula(c(0.6,0.7,0.6,0.8), rf)
C.n(cbind(0.6,0.7,0.6,0.8), u)
```

```
## Fitting
(rf. <- rotCopula(frankCopula(dim=4),
                 flip = c(TRUE, FALSE, TRUE, FALSE)))
## fitCopula(rf., data = pobs(u), method = "irho")
## FIXME above: not related to rotCopula but frankCopula
fitCopula(rf., data = pobs(u), method = "itau")
fitCopula(rf., data = pobs(u), method = "mpl")

## Goodness-of-fit testing (first ("PB") is really slow, see above):
## Not run: gofCopula(rf., x = u)
gofCopula(rf., x = u, simulation = "mult") # takes 3.7 sec [lynne, 2015]
```

---

|        |                                                                                             |
|--------|---------------------------------------------------------------------------------------------|
| RSpobs | <i>Pseudo-Observations of Radial and Uniform Part of Elliptical and Archimedean Copulas</i> |
|--------|---------------------------------------------------------------------------------------------|

---

## Description

Given a matrix of iid multivariate data from a meta-elliptical or meta-Archimedean model, `RSpobs()` computes pseudo-observations of the radial part  $R$  and the vector  $\mathbf{S}$  which follows a uniform distribution on the unit sphere (for elliptical copulas) or the unit simplex (for Archimedean copulas). These quantities can be used for (graphical) goodness-of-fit tests, for example.

## Usage

```
RSpobs(x, do.pobs = TRUE, method = c("ellip", "archm"), ...)
```

## Arguments

|                      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |
|----------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>x</code>       | an $(n, d)$ -matrix of data; if <code>do.pobs=FALSE</code> , the rows of <code>x</code> are assumed to lie in the $d$ -dimensional unit hypercube (if they do not, this leads to an error).                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| <code>do.pobs</code> | <b>logical</b> indicating whether <code>pobs()</code> is applied to <code>x</code> for transforming the data to the $d$ -dimensional unit hypercube.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
| <code>method</code>  | <b>character</b> string indicating the assumed underlying model, being meta-elliptical if <code>method="ellip"</code> (in which case $\mathbf{S}$ should be approximately uniform on the $d$ -dimensional unit sphere) or meta-Archimedean if <code>method="archm"</code> (in which case $\mathbf{S}$ should be approximately uniform on the $d$ -dimensional unit simplex).                                                                                                                                                                                                                                                                                |
| <code>...</code>     | additional arguments passed to the implemented methods. These can be <code>method="ellip"</code> <code>qQg()</code> (the quantile function of the (assumed) distribution function $G_g$ as given in Genest, Hofert, G. Nešlehová (2014)); if provided, <code>qQg()</code> is used in the transformation for obtaining pseudo-observations of $R$ and $\mathbf{S}$ (see the code for more details).<br><code>method="archm"</code> <code>iPsi()</code> (the assumed underlying generator inverse); if provided, <code>iPsi()</code> is used in the transformation for obtaining pseudo-observations of $R$ and $\mathbf{S}$ (see the code for more details). |

## Details

The construction of the pseudo-observations of the radial part and the uniform distribution on the unit sphere/simplex is described in Genest, Hofert, G. Nešlehová (2014).

## Value

A `list` with components `R` (an  $n$ -vector containing the pseudo-observations of the radial part) and `S` (an  $(n, d)$ -matrix containing the pseudo-observations of the uniform distribution (on the unit sphere/simplex)).

## References

Genest, C., Hofert, M., G. Nešlehová, J., (2014). Is the dependence Archimedean, elliptical, or what? *To be submitted*.

## See Also

`pobs()` for computing the “classical” pseudo-observations.

## Examples

```
set.seed(100)
n <- 250 # sample size
d <- 5 # dimension
nu <- 3 # degrees of freedom

## Build a mean vector and a dispersion matrix,
## and generate multivariate t_nu data:
mu <- rev(seq_len(d)) # d, d-1, ..., 1
L <- diag(d) # identity in dim d
L[lower.tri(L)] <- 1:(d*(d-1)/2)/d # Cholesky factor (diagonal > 0)
Sigma <- crossprod(L) # pos.-def. dispersion matrix (*not* covariance of X)
X <- rep(mu, each=n) + mvtnorm::rmvt(n, sigma=Sigma, df=nu) # multiv. t_nu data
## note: this is *wrong*: mvtnorm::rmvt(n, mean=mu, sigma=Sigma, df=nu)

## compute pseudo-observations of the radial part and uniform distribution
## once for 1a), once for 1b) below
RS.t <- RSpobs(X, method="ellip", qQg=function(p) qt(p, df=nu)) # 'correct'
RS.norm <- RSpobs(X, method="ellip", qQg=qnorm) # for testing 'wrong' distribution
stopifnot(length(RS.norm$R) == n, length(RS.t$R) == n,
           dim(RS.norm$S) == c(n,d), dim(RS.t$S) == c(n,d))

## 1) Graphically testing the radial part

## 1a) Q-Q plot of R against the correct quantiles
qqplot2(RS.t$R, qF=function(p) sqrt(d*qf(p, df1=d, df2=nu)),
        main.args=list(text= substitute(bold(italic(F[list(d.,nu.)](r^2/d.))~~"Q-Q Plot")),
                    list(d.=d, nu.=nu)),
        side=3, cex=1.3, line=1.1, xpd=NA))

## 1b) Q-Q plot of R against the quantiles of F_R for a multivariate normal
```

```

##      distribution
qqplot2(RS.norm$R, qF=function(p) sqrt(qchisq(p, df=d)),
        main.args=list(text= substitute(italic(chi[D_]) ~ "Q-Q Plot"), list(D_=d)),
        side=3, cex=1.3, line=1.1, xpd=NA))

## 2) Graphically testing the angular distribution

## auxiliary function
qqp <- function(k, Bmat) {
  d <- ncol(Bmat) + 1
  qqplot2(Bmat[,k],
          qF = function(p) qbeta(p, shape1=k/2, shape2=(d-k)/2),
          main.args=list(text= substitute(plain("Beta")(s1,s2) ~ bold("Q-Q Plot"),
                                     list(s1 = k/2, s2 = (d-k)/2)),
                        side=3, cex=1.3, line=1.1, xpd=NA))
}

## 2a) Q-Q plot of the 'correct' angular distribution
##      (Bmat[,k] should follow a Beta(k/2, (d-k)/2) distribution)
Bmat.t <- gofBTstat(RS.t$S)
qqp(1, Bmat=Bmat.t) # k=1
qqp(3, Bmat=Bmat.t) # k=3

## 2b) Q-Q plot of the 'wrong' angular distribution
Bmat.norm <- gofBTstat(RS.norm$S)
qqp(1, Bmat=Bmat.norm) # k=1
qqp(3, Bmat=Bmat.norm) # k=3

## 3) Graphically check independence between radial part and B_1 and B_3

## 'correct' distributions (multivariate t)
plot(pobs(cbind(RS.t$R, Bmat.t[,1])), # k = 1
     xlab=quote(italic(R)), ylab=quote(italic(B)[1]),
     main=quote(bold("Rank plot between" ~ italic(R) ~ "and" ~ italic(B)[1])))
plot(pobs(cbind(RS.t$R, Bmat.t[,3])), # k = 3
     xlab=quote(italic(R)), ylab=quote(italic(B)[3]),
     main=quote(bold("Rank plot between" ~ italic(R) ~ "and" ~ italic(B)[3])))

## 'wrong' distributions (multivariate normal)
plot(pobs(cbind(RS.norm$R, Bmat.norm[,1])), # k = 1
     xlab=quote(italic(R)), ylab=quote(italic(B)[1]),
     main=quote(bold("Rank plot between" ~ italic(R) ~ "and" ~ italic(B)[1])))
plot(pobs(cbind(RS.norm$R, Bmat.norm[,3])), # k = 3
     xlab=quote(italic(R)), ylab=quote(italic(B)[3]),
     main=quote(bold("Rank plot between" ~ italic(R) ~ "and" ~ italic(B)[3])))

```

**Description**

Generate random numbers of the stable distribution

$$S(\alpha, \beta, \gamma, \delta; k)$$

with characteristic exponent  $\alpha \in (0, 2]$ , skewness  $\beta \in [-1, 1]$ , scale  $\gamma \in [0, \infty)$ , and location  $\delta \in \mathbf{R}$ ; see Nolan (2010) for the parameterization  $k \in \{0, 1\}$ . The case  $\gamma = 0$  is understood as the unit jump at  $\delta$ .

**Usage**

```
rstable1(n, alpha, beta, gamma = 1, delta = 0, pm = 1)
```

**Arguments**

|       |                                                                      |
|-------|----------------------------------------------------------------------|
| n     | an <a href="#">integer</a> , the number of observations to generate. |
| alpha | characteristic exponent $\alpha \in (0, 2]$ .                        |
| beta  | skewness $\beta \in [-1, 1]$ .                                       |
| gamma | scale $\gamma \in [0, \infty)$ .                                     |
| delta | location $\delta \in \mathbf{R}$ .                                   |
| pm    | 0 or 1, denoting which parametrization (as by Nolan) is used.        |

**Details**

We use the approach of John Nolan for generating random variates of stable distributions. The function `rstable1` provides two basic parametrizations, by default,

`pm = 1`, the so called “S”, “S1”, or “1” parameterization. This is the parameterization used by Samorodnitsky and Taqqu (1994), and is a slight modification of Zolotarev’s (A) parameterization. It is the form with the most simple form of the characteristic function; see Nolan (2010, p. 8).

`pm = 0` is the “S0” parameterization: based on the (M) representation of Zolotarev for an alpha stable distribution with skewness beta. Unlike the Zolotarev (M) parameterization, gamma and delta are straightforward scale and shift parameters. This representation is continuous in all 4 parameters.

**Value**

A [numeric](#) vector of length n containing the generated random variates.

**References**

Chambers, J. M., Mallows, C. L., and Stuck, B. W. (1976), *A Method for Simulating Stable Random Variables*, J. Amer. Statist. Assoc. **71**, 340–344.

Nolan, J. P. (2012), *Stable Distributions—Models for Heavy Tailed Data*, Birkhaeuser, in progress.

Samoridnitsky, G. and Taqqu, M. S. (1994), *Stable Non-Gaussian Random Processes, Stochastic Models with Infinite Variance*, Chapman and Hall, New York.



**See Also**

[rstable](#) which also allows the 2-parametrization and provides further functionality for stable distributions.

**Examples**

```
# Generate and plot a series of stable random variates
set.seed(1953)
r <- rstable1(n = 1000, alpha = 1.9, beta = 0.3)
plot(r, type = "l", main = "stable: alpha=1.9 beta=0.3",
     col = "steelblue"); grid()

hist(r, "Scott", prob = TRUE, ylim = c(0,0.3),
     main = "Stable S(1.9, 0.3; 1)")
lines(density(r), col="red2", lwd = 2)
```

---

safeUroot

*One-dimensional Root (Zero) Finding - Extra "Safety" for Convenience*


---

**Description**

safeUroot() as a “safe” version of [uniroot\(\)](#) searches for a root (that is, zero) of the function *f* with respect to its first argument.

“Safe” means searching for the correct interval = c(lower, upper) if sign(*f*(*x*)) does not satisfy the requirements at the interval end points; see the ‘Details’ section.

**Usage**

```
safeUroot(f, interval, ...,
          lower = min(interval), upper = max(interval),
          f.lower = f(lower, ...), f.upper = f(upper, ...),
          Sig = NULL, check.conv = FALSE,
          tol = .Machine$double.eps^0.25, maxiter = 1000, trace = 0)
```

**Arguments**

|                                 |                                                                                                                          |
|---------------------------------|--------------------------------------------------------------------------------------------------------------------------|
| <i>f</i>                        | function                                                                                                                 |
| <i>interval</i>                 | interval                                                                                                                 |
| ...                             | additional named or unnamed arguments to be passed to <i>f</i>                                                           |
| <i>lower</i> , <i>upper</i>     | lower and upper endpoint of search interval                                                                              |
| <i>f.lower</i> , <i>f.upper</i> | function value at lower or upper endpoint, respectively.                                                                 |
| <i>Sig</i>                      | <i>desired</i> sign of <i>f</i> ( <i>upper</i> ), or <b>NULL</b> .                                                       |
| <i>check.conv</i>               | logical indicating whether a convergence warning of the underlying <a href="#">uniroot</a> should be caught as an error. |

|         |                                                       |
|---------|-------------------------------------------------------|
| tol     | the desired accuracy, that is, convergence tolerance. |
| maxiter | maximal number of iterations                          |
| trace   | number determining tracing                            |

### Details

If it is *known how*  $f$  changes sign at the root  $x_0$ , that is, if the function is increasing or decreasing there, `Sig` can be specified, typically as  $S := \pm 1$ , to require  $S = \text{sign}(f(x_0 + \epsilon))$  at the solution. In that case, the search interval  $[l, u]$  must be such that  $S * f(l) \leq 0$  and  $S * f(u) \geq 0$ .

Otherwise, by default, when `Sig=NULL`, the search interval  $[l, u]$  must satisfy  $f(l) * f(u) \leq 0$ .

In both cases, when the requirement is not satisfied, `safeUroot()` tries to enlarge the interval until the requirement *is* satisfied.

### Value

A list with four components, `root`, `f.root`, `iter` and `estim.prec`; see [uniroot](#).

### See Also

[uniroot](#).

### Examples

```
f1 <- function(x) (121 - x^2)/(x^2+1)
f2 <- function(x) exp(-x)*(x - 12)

try(uniroot(f1, c(0,10)))
try(uniroot(f2, c(0,2)))
##--> error: f() .. end points not of opposite sign

## where as safeUroot() simply first enlarges the search interval:
safeUroot(f1, c(0,10),trace=1)
safeUroot(f2, c(0,2), trace=2)

## no way to find a zero of a positive function:
try( safeUroot(exp, c(0,2), trace=TRUE) )

## Convergence checking :
safeUroot(sinc, c(0,5), maxiter=4) #-> "just" a warning
try( # an error, now with check.conv=TRUE
  safeUroot(sinc, c(0,5), maxiter=4, check.conv=TRUE) )
```

---

|                 |                                                                                 |
|-----------------|---------------------------------------------------------------------------------|
| serialIndepTest | <i>Serial Independence Test for Continuous Time Series Via Empirical Copula</i> |
|-----------------|---------------------------------------------------------------------------------|

---

### Description

Computes the serial independence test based on the empirical copula process as proposed in Ghoudi et al.(2001) and Genest and Rémillard (2004). The test, which is the serial analog of [indepTest](#), can be seen as composed of three steps:

- (i) a simulation step, which consists in simulating the distribution of the test statistics under serial independence for the sample size under consideration;
- (ii) the test itself, which consists in computing the approximate p-values of the test statistics with respect to the empirical distributions obtained in step (i);
- (iii) the display of a graphic, called a *dependogram*, enabling to understand the type of departure from serial independence, if any.

More details can be found in the articles cited in the reference section.

### Usage

```
serialIndepTestSim(n, lag.max, m=lag.max+1, N=1000, verbose = interactive())
serialIndepTest(x, d, alpha=0.05)
```

### Arguments

|         |                                                                                                                                                                                                                   |
|---------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| n       | length of the time series when simulating the distribution of the test statistics under serial independence.                                                                                                      |
| lag.max | maximum lag.                                                                                                                                                                                                      |
| m       | maximum cardinality of the subsets of 'lags' for which a test statistic is to be computed. It makes sense to consider $m \ll \text{lag.max}+1$ especially when $\text{lag.max}$ is large.                         |
| N       | number of repetitions when simulating under serial independence.                                                                                                                                                  |
| verbose | a logical specifying if progress should be displayed via <a href="#">txtProgressBar</a> .                                                                                                                         |
| x       | numeric vector containing the time series whose serial independence is to be tested.                                                                                                                              |
| d       | object of class <code>serialIndepTestDist</code> as returned by the function <code>serialIndepTestSim</code> . It can be regarded as the empirical distribution of the test statistics under serial independence. |
| alpha   | significance level used in the computation of the critical values for the test statistics.                                                                                                                        |

### Details

See the references below for more details, especially the third and fourth ones.

**Value**

The function `serialIndepTestSim()` returns an object of S3 class "serialIndepTestDist" with list components `sample.size`, `lag.max`, `max.card.subsets`, `number.repetitions`, `subsets` (list of the subsets for which test statistics have been computed), `subsets.binary` (subsets in binary 'integer' notation), `dist.statistics.independence` (a N line matrix containing the values of the test statistics for each subset and each repetition) and `dist.global.statistic.independence` (a vector a length N containing the values of the serial version of the global Cramér-von Mises test statistic for each repetition — see last reference p.175).

The function `serialIndepTest()` returns an object of S3 class "indepTest" with list components `subsets`, `statistics`, `critical.values`, `pvalues`, `fisher.pvalue` (a p-value resulting from a combination *à la* Fisher of the subset statistic p-values), `tippett.pvalue` (a p-value resulting from a combination *à la* Tippett of the subset statistic p-values), `alpha` (global significance level of the test), `beta` ( $1 - \beta$  is the significance level per statistic), `global.statistic` (value of the global Cramér-von Mises statistic derived directly from the serial independence empirical copula process — see last reference p 175) and `global.statistic.pvalue` (corresponding p-value).

The former argument `print.every` is deprecated and not supported anymore; use `verbose` instead.

**References**

Deheuvels, P. (1979). La fonction de dépendance empirique et ses propriétés: un test non paramétrique d'indépendance, *Acad. Roy. Belg. Bull. Cl. Sci.*, 5th Ser. 65:274–292.

Deheuvels, P. (1981), A non parametric test for independence, *Publ. Inst. Statist. Univ. Paris*. 26:29–50.

Genest, C. and Rémillard, B. (2004) Tests of independence and randomness based on the empirical copula process. *Test* **13**, 335–369.

Genest, C., Quessy, J.-F., and Rémillard, B. (2006) Local efficiency of a Cramer-von Mises test of independence. *Journal of Multivariate Analysis* **97**, 274–294.

Genest, C., Quessy, J.-F., and Rémillard, B. (2007) Asymptotic local efficiency of Cramér-von Mises tests for multivariate independence. *The Annals of Statistics* **35**, 166–191.

**See Also**

[indepTest](#), [multIndepTest](#), [multSerialIndepTest](#), [dependogram](#)

**Examples**

```
## AR 1 process

ar <- numeric(200)
ar[1] <- rnorm(1)
for (i in 2:200)
  ar[i] <- 0.5 * ar[i-1] + rnorm(1)
x <- ar[101:200]

## In order to test for serial independence, the first step consists
## in simulating the distribution of the test statistics under
## serial independence for the same sample size, i.e. n=100.
## As we are going to consider lags up to 3, i.e., subsets of
```

```

## {1,...,4} whose cardinality is between 2 and 4 containing {1},
## we set lag.max=3. This may take a while...

d <- serialIndepTestSim(100,3)

## The next step consists in performing the test itself:

test <- serialIndepTest(x,d)

## Let us see the results:

test

## Display the dependogram:

dependogram(test,print=TRUE)

## NB: In order to save d for future use, the saveRDS() function can be used.

```

---

setTheta

*Specify the Parameter(s) of a Copula*


---

### Description

Set or change the parameter  $\theta$  (theta) of a copula. The name ‘theta’ has been from its use in (nested) Archimedean copulas, where  $x$  is of class “[acopula](#)” or “[outer\\_nacopula](#)”. This is used for constructing copula models with specified parameter, as, for example, in [onacopula\(\)](#), or also [gofCopula](#).

### Usage

```

setTheta(x, value, na.ok = TRUE, noCheck = FALSE, freeOnly = TRUE, ...)

## S4 method for signature 'acopula,ANY'
setTheta(x, value, na.ok = TRUE, noCheck = FALSE, freeOnly = TRUE, ...)
## S4 method for signature 'copula,ANY'
setTheta(x, value, na.ok = TRUE, noCheck = FALSE, freeOnly = TRUE, ...)
## S4 method for signature 'Xcopula,ANY'
setTheta(x, value, na.ok = TRUE, noCheck = FALSE, freeOnly = TRUE, ...)
## S4 method for signature 'outer_nacopula,numeric'
setTheta(x, value, na.ok = TRUE, noCheck = FALSE, freeOnly = TRUE, ...)
## S4 method for signature 'khoudrajiCopula,ANY'
setTheta(x, value, na.ok = TRUE, noCheck = FALSE, freeOnly = TRUE, ...)

## S4 method for signature 'mixCopula,ANY'
setTheta(x, value, na.ok = TRUE, noCheck = FALSE, freeOnly = TRUE,
        treat.negative = c("set.0", "warn.set0", "stop"), ...)

```

**Arguments**

|                |                                                                                                                                                                                                              |
|----------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| x              | an R object of class <code>Copula</code> , i.e., any copula from package <code>copula</code> .                                                                                                               |
| value          | parameter value or vector, <code>numeric</code> or <code>NA</code> (when <code>na.ok</code> is true), corresponding to the “free” parameters.                                                                |
| ...            | further arguments for methods.                                                                                                                                                                               |
| na.ok          | logical indicating if <code>NA</code> values are ok for theta.                                                                                                                                               |
| noCheck        | logical indicating if parameter constraint checks should be skipped.                                                                                                                                         |
| freeOnly       | logical indicating that only non-fixed aka “free” parameters are to be set. If true as by default, <code>setTheta()</code> modifies only the free parameters of the copula; see also <code>fixParam</code> . |
| treat.negative | a <code>character</code> string indicating how negative mixture weights should be handled. If not “stop” which produces an error via <code>stop</code> , negative mixture weights are replaced by zero.      |

**Value**

an R object of the same class as `x`, with the main parameter (vector) (often called `theta`) set to `value`.

**See Also**

the “inverse” function, a “getter” method, is `getTheta()`.

**Examples**

```
myC <- setTheta(copClayton, 0.5)
myC
## Frank copula with Kendall's tau = 0.8 :
(myF.8 <- setTheta(copFrank, iTau(copFrank, tau = 0.8)))

# negative theta is ok for dim = 2 :
myF <- setTheta(copFrank, -2.5, noCheck=TRUE)
myF@tau(myF@theta) # -0.262

myT <- setTheta(tCopula(df.fixed=TRUE), 0.7)
stopifnot(all.equal(myT, tCopula(0.7, df.fixed=TRUE),
                    check.environment=FALSE, tolerance=0))

(myT2 <- setTheta(tCopula(dim=3, df.fixed=TRUE), 0.7))
## Setting 'rho' and 'df' --- for default df.fixed=FALSE :
(myT3 <- setTheta(tCopula(dim=3), c(0.7, 4)))
```

---

 show-methods

 Methods for 'show()' in Package 'copula'
 

---

### Description

Methods for function `show` in package `copula`.

### Methods

**object = "parCopula"** see `Copula`.

**object = "fitMvdc"** (see `fitMvdc`): and

**object = "fitCopula"** (see `fitCopula`): these call the (hidden) `print` method, with its default argument. Using `print()` instead, allows to set digits, e.g.

**object = "fitMvdc"** see `fitCopula`.

---

Sibuya

*Sibuya Distribution - Sampling and Probabilities*


---

### Description

The Sibuya distribution  $\text{Sib}(\alpha)$  can be defined by its Laplace transform

$$1 - (1 - \exp(-t))^\alpha, \quad t \in [0, \infty),$$

its distribution function

$$F(k) = 1 - (-1)^k \binom{\alpha - 1}{k} = 1 - \frac{1}{kB(k, 1 - \alpha)}, \quad k \in \mathbf{N}$$

(where  $B$  denotes the beta function) or its probability mass function

$$p_k = \binom{\alpha}{k} (-1)^{k-1}, \quad k \in \mathbf{N},$$

where  $\alpha \in (0, 1]$ .

`pSibuya` evaluates the distribution function.

`dSibuya` evaluates the probability mass function.

`rSibuya` generates random variates from  $\text{Sib}(\alpha)$  with the algorithm described in Hofert (2011), Proposition 3.2.

`dsumSibuya` gives the probability mass function of the  $n$ -fold convolution of Sibuya variables, that is, the sum of  $n$  independent Sibuya random variables,  $S = \sum_{i=1}^n X_i$ , where  $X_i \sim \text{Sib}(\alpha)$ .

This probability mass function can be shown (see Hofert (2010, pp. 99)) to be

$$\sum_{j=1}^n \binom{n}{j} \binom{j\alpha}{k} (-1)^{k-j}, \quad k \in \{n, n+1, \dots\}.$$

**Usage**

```

rSibuya(n, alpha)
dSibuya(x, alpha, log=FALSE)
pSibuya(x, alpha, lower.tail=TRUE, log.p=FALSE)

dsumSibuya(x, n, alpha,
           method=c("log", "direct", "diff", "exp.log",
                    "Rmpfr", "Rmpfr0", "RmpfrM", "Rmpfr0M"),
           mpfr.ctrl = list(minPrec = 21, fac = 1.25, verbose=TRUE),
           log=FALSE)

```

**Arguments**

**n** for `rSibuya`: sample size, that is, length of the resulting vector of random variates.  
for `dsumSibuya`: the number  $n$  of summands.

**alpha** parameter in  $(0, 1]$ .

**x** vector of `integer` values (“quantiles”)  $x$  at which to compute the probability mass or cumulative probability.

**log, log.p** `logical`; if `TRUE`, probabilities  $p$  are given as  $\log(p)$ .

**lower.tail** `logical`; if `TRUE` (the default), probabilities are  $P(X \leq x)$ , otherwise,  $P(X > x)$ .

**method** character string specifying which computational method is to be applied. Implemented are:

“log” evaluates the logarithm of the sum

$$\sum_{j=1}^n \binom{n}{j} \binom{j\alpha}{x} (-1)^{x-j}$$

in a numerically stable way;

“direct” directly evaluates the sum;

“Rmpfr\*” are as `method="direct"` but use high-precision arithmetic; “Rmpfr” and “Rmpfr0” return `doubles` whereas “RmpfrM” and “Rmpfr0M” give `mpfr` high-precision numbers. Whereas “Rmpfr” and “RmpfrM” each adapt to high enough precision, the “Rmpfr0\*” ones do not adapt.

For all “Rmpfr\*” methods, `alpha` can be set to a `mpfr` number of specified precision and this will determine the precision of all parts of the internal computations.

“diff” interprets the sum as a forward difference and computes it via `diff`;

“exp.log” is as `method="log"` but without numerically stable evaluation (not recommended, use with care).

**mpfr.ctrl** for `method = "Rmpfr"` or “RmpfrM” only: a list of  
**minPrec**: minimal (estimated) precision in bits,  
**fac**: factor with which current precision is multiplied if it is not sufficient.  
**verbose**: determining if and how much is printed.



### Details

The Sibuya distribution has **no** finite moments, that is, specifically infinite mean and variance.

For documentation and didactical purposes, `rSibuyaR` is a pure-R implementation of `rSibuya`, of course slower than `rSibuya` as the latter is implemented in C.

Note that the sum to evaluate for `dsumSibuya` is numerically highly challenging, even already for small  $\alpha$  values (for example,  $n \geq 10$ ), and therefore should be used with care. It may require high-precision arithmetic which can be accessed with `method="Rmpfr"` (and the **Rmpfr** package).

### Value

**rSibuya:** A vector of positive [integers](#) of length `n` containing the generated random variates.

**dSibuya, pSibuya:** a vector of probabilities of the same length as `x`.

**dsumSibuya:** a vector of probabilities, positive if and only if `x >= n` and of the same length as `x` (or `n` if that is longer).

### References

Hofert, M. (2010). *Sampling Nested Archimedean Copulas with Applications to CDO Pricing*. Südwestdeutscher Verlag fuer Hochschulschriften AG & Co. KG.

Hofert, M. (2011). Efficiently sampling nested Archimedean copulas. *Computational Statistics & Data Analysis* **55**, 57–70.

### See Also

[rFJoe](#) and [rF01Joe](#) (where `rSibuya` is applied).

### Examples

```
## Sample n random variates from a Sibuya(alpha) distribution and plot a
## histogram
n <- 1000
alpha <- .4
X <- rSibuya(n, alpha)
hist(log(X), prob=TRUE); lines(density(log(X)), col=2, lwd=2)
```

### Description

`SMI.12` contains the close prices of all 20 constituents of the Swiss Market Index (SMI) from 2011-09-09 to 2012-03-28.

### Usage

```
data(SMI.12)
```

## Format

SMI.12 is conceptually a multivariate time series, here simply stored as `numeric matrix`, where the `rownames` are dates (of week days).

The format is:

```
num [1:141, 1:20] 16.1 15.7 15.7 16.1 16.6 ... - attr(*, "dimnames")=List of 2 ..$ : chr [1:141]
"2011-09-09" "2011-09-12" "2011-09-13" "2011-09-14" ... ..$ : chr [1:20] "ABBN" "ATLN"
"ADEN" "CSGN" ...
```

... from 2011-09-09 to 2012-03-28

1SMI is the list of the original data (*before* NA “imputation”).

## Source

The data was drawn from Yahoo! Finance.

## Examples

```
data(SMI.12)
## maybe
head(SMI.12)

str(D.12 <- as.Date(rownames(SMI.12)))
summary(D.12)

matplot(D.12, SMI.12, type="l", log = "y",
        main = "The 20 SMI constituents (2011-09 -- 2012-03)",
        xaxt="n", xlab = "2011 / 2012")
Axis(D, side=1)

if(FALSE) { ##--- This worked up to mid 2012, but no longer ---
  begSMI <- "2011-09-09"
  endSMI <- "2012-03-28"
  ##-- read *public* data -----
  stopifnot(require(zoo), # -> to access all the zoo methods
            require(tseries))
  symSMI <- c("ABBN.VX", "ATLN.VX", "ADEN.VX", "CSGN.VX", "GIVN.VX", "HOLN.VX",
            "BAER.VX", "NESN.VX", "NOVN.VX", "CFR.VX", "ROG.VX", "SGSN.VX",
            "UHR.VX", "SREN.VX", "SCMN.VX", "SYNN.VX", "SYST.VX", "RIGN.VX",
            "UBSN.VX", "ZURN.VX")
  1SMI <- sapply(symSMI, function(sym)
    get.hist.quote(instrument = sym, start= begSMI, end= endSMI,
                  quote = "Close", provider = "yahoo",
                  drop=TRUE))
  ## check if stock data have the same length for each company.
  sapply(1SMI, length)
  ## "concatenate" all:
  SMIo <- do.call(cbind, 1SMI)
  ## and fill in the NAs :
  SMI.12 <- na.fill(SMIo, "extend")
}
```

```

colnames(SMI.12) <- sub("\\.VX", "", colnames(SMI.12))
SMI.12 <- as.matrix(SMI.12)
}##----- --- original download

zoo.there <- "package:zoo" %in% search()
if(zoo.there || require("zoo")) {
  stopifnot(identical(SMI.12,
    local({ S <- as.matrix(na.fill(do.call(cbind, lSMI), "extend"))
      colnames(S) <- sub("\\.VX", "", colnames(S)); S })))
  if(!zoo.there) detach("package:zoo")
}

```

splom2-methods

*Methods for Scatter Plot Matrix 'splom2' in Package 'copula'***Description**

Methods `splom2()` to draw scatter-plot matrices of (random samples of) distributions from package **copula**.

**Usage**

```

## S4 method for signature 'matrix'
splom2(x, varnames = NULL, varnames.null.lab = "U",
       xlab = "", col.mat = NULL, bg.col.mat = NULL, ...)
## ditto an identical 'data.frame' method

## S4 method for signature 'Copula'
splom2(x, n, ...)
## S4 method for signature 'mvdc'
splom2(x, n, varnames.null.lab = "X", ...)

```

**Arguments**

|                                |                                                                                                                                                                                                                                                                        |
|--------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>x</code>                 | a "matrix", "data.frame", "Copula" or a "mvdc" object.                                                                                                                                                                                                                 |
| <code>n</code>                 | when <code>x</code> is not matrix-like: The sample size of the random sample drawn from <code>x</code> .                                                                                                                                                               |
| <code>varnames</code>          | the variable names, typically unspecified.                                                                                                                                                                                                                             |
| <code>varnames.null.lab</code> | the <a href="#">character</a> string determining the "base name" of the variable labels in case <code>varnames</code> is <code>NULL</code> and <code>x</code> does not have all column names given.                                                                    |
| <code>xlab</code>              | the x-axis label.                                                                                                                                                                                                                                                      |
| <code>col.mat</code>           | a matrix of colors (or one color) for the plot symbols; if <code>NULL</code> (as by default), <a href="#">trellis.par.get("plot.symbol")\$col</a> is used for all symbols. (Note that in <b>copula</b> version 0.999-15, this was not true; instead "black" was used.) |
| <code>bg.col.mat</code>        | a matrix of colors for the background (the default is the setting as obtained from <a href="#">trellis.par.get("background")\$col</a> ).                                                                                                                               |
| <code>...</code>               | additional arguments passed to the underlying <a href="#">splom()</a> .                                                                                                                                                                                                |

**Value**

From `splom()`, an R object of class "trellis".

**See Also**

`pairs2()` for a similar function (for matrices and data frames) based on `pairs()`.

The **lattice**-based `cloud2-methods` for 3D data, and `wireframe2-methods` and `contourplot2-methods` for functions.

**Examples**

```
## For 'matrix' objects
## Create a 100 x 7 matrix of random variates from a t distribution
## with four degrees of freedom and plot the generated data
n <- 1000 # sample size
d <- 3 # dimension
nu <- 4 # degrees of freedom
tau <- 0.5 # Kendall's tau
th <- iTau(tCopula(df = nu), tau) # corresponding parameter
cop <- tCopula(th, dim = d, df = nu) # define copula object
set.seed(271)
U <- rCopula(n, copula = cop)
splom2(U)

## For 'copula' objects
set.seed(271)
splom2(cop, n = n) # same as above

## For 'rotCopula' objects: ---> Examples in rotCopula

## For 'mvdc' objects
mvNN <- mvdc(cop, c("norm", "norm", "exp"),
             list(list(mean = 0, sd = 1), list(mean = 1), list(rate = 2)))
splom2(mvNN, n = n)
```

---

Stirling

*Eulerian and Stirling Numbers of First and Second Kind*

---

**Description**

Compute Eulerian numbers and Stirling numbers of the first and second kind, possibly vectorized for all  $k$  "at once".

**Usage**

```
Stirling1(n, k)
Stirling2(n, k, method = c("lookup.or.store", "direct"))
Eulerian (n, k, method = c("lookup.or.store", "direct"))
```

```
Stirling1.all(n)
Stirling2.all(n)
Eulerian.all (n)
```

### Arguments

|        |                                                                                                                                                                     |
|--------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| n      | positive integer (0 is allowed for Eulerian()).                                                                                                                     |
| k      | integer in 0:n.                                                                                                                                                     |
| method | for Eulerian() and Stirling2(), string specifying the method to be used. "direct" uses the explicit formula (which may suffer from some cancelation for "large" n). |

### Details

Eulerian numbers:

$A(n, k)$  = the number of permutations of 1,2,..,n with exactly  $k$  ascents (or exactly  $k$  descents).

Stirling numbers of the first kind:

$s(n, k) = (-1)^{n-k}$  times the number of permutations of 1,2,..,n with exactly  $k$  cycles.

Stirling numbers of the second kind:

$S_n^{(k)}$  is the number of ways of partitioning a set of  $n$  elements into  $k$  non-empty subsets.

### Value

$A(n, k)$ ,  $s(n, k)$  or  $S(n, k) = S_n^{(k)}$ , respectively.

Eulerian.all(n) is the same as sapply(0:(n-1), Eulerian, n=n) (for  $n > 0$ ),

Stirling1.all(n) is the same as sapply(1:n, Stirling1, n=n), and

Stirling2.all(n) is the same as sapply(1:n, Stirling2, n=n), but more efficient.

### Note

For typical double precision arithmetic,

Eulerian\*(n, \*) overflow (to Inf) for  $n \geq 172$ ,

Stirling1\*(n, \*) overflow (to  $\pm$ Inf) for  $n \geq 171$ , and

Stirling2\*(n, \*) overflow (to Inf) for  $n \geq 220$ .

### References

#### Eulerians:

NIST Digital Library of Mathematical Functions, 26.14: <https://dlmf.nist.gov/26.14>

#### Stirling numbers:

Abramowitz and Stegun 24,1,4 (p. 824-5 ; Table 24.4, p.835); Closed Form : p.824 "C."

NIST Digital Library of Mathematical Functions, 26.8: <https://dlmf.nist.gov/26.8>

**Examples**

```

Stirling1(7,2)
Stirling2(7,3)

Stirling1.all(9)
Stirling2.all(9)

```

---

tauAMH

*Ali-Mikhail-Haq ("AMH")'s and Joe's Kendall's Tau*


---

**Description**

Compute Kendall's Tau of an Ali-Mikhail-Haq ("AMH") or Joe Archimedean copula with parameter  $\theta$ . In both cases, analytical expressions are available, but need alternatives in some cases.

tauAMH(): Analytically, given as

$$1 - \frac{2((1 - \theta)^2 \log(1 - \theta) + \theta)}{3\theta^2},$$

for  $\theta = \theta$ ; numerically, care has to be taken when  $\theta \rightarrow 0$ , avoiding accuracy loss already, for example, for  $\theta$  as large as  $\theta = 0.001$ .

tauJoe(): Analytically,

$$1 - 4 \sum_{k=1}^{\infty} \frac{1}{k(\theta k + 2)(\theta(k - 1) + 2)},$$

the infinite sum can be expressed by three  $\psi()$  ([psigamma](#)) function terms.

**Usage**

```

tauAMH(theta)
tauJoe(theta, method = c("hybrid", "digamma", "sum"), noTerms=446)

```

**Arguments**

|         |                                                                                                                                                                              |
|---------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| theta   | numeric vector with values in $[-1, 1]$ for AMH, or $[0.238734, Inf)$ for Joe.                                                                                               |
| method  | string specifying the method for tauJoe(). Use the default, unless for research about the method. Up to <b>copula</b> version 0.999-0, the only (implicit) method was "sum". |
| noTerms | the number of summation terms for the "sum" method; its default, 446 gives an absolute error smaller than $10^{-5}$ .                                                        |

## Details

tauAMH(): For small theta ( $= \theta$ ), we use Taylor series approximations of up to order 7,

$$\tau_A(\theta) = \frac{2}{9}\theta\left(1 + \theta\left(\frac{1}{4} + \frac{\theta}{10}\left(1 + \theta\left(\frac{1}{2} + \theta\frac{2}{7}\right)\right)\right)\right) + O(\theta^6),$$

where we found that dropping the last two terms (e.g., only using 5 terms from the  $k = 7$  term Taylor polynomial) is actually numerically advantageous.

tauJoe(): The "sum" method simply replaces the infinite sum by a finite sum (with noTerms terms. The more accurate or faster methods, use analytical summation formulas, using the [digamma](https://en.wikipedia.org/wiki/Digamma_function#Series_formula) aka  $\psi$  function, see, e.g., [https://en.wikipedia.org/wiki/Digamma\\_function#Series\\_formula](https://en.wikipedia.org/wiki/Digamma_function#Series_formula).

The smallest sensible  $\theta$  value, i.e., th for which tauJoe(th) == -1 is easily determined via str(uniroot(function(th) tauJoe(th)-(-1), c(0.1, 0.3), tol = 1e-17), digits=12) to be 0.2387339899.

## Value

a vector of the same length as theta ( $= \theta$ ), with  $\tau$  values

for tauAMH: in  $[(5 - 8\log 2)/3, 1/3] = [-0.1817, 0.3333]$ , of  $\tau_A(\theta) = 1 - 2(\theta + (1 - \theta)^2 \log(1 - \theta))/(3\theta^2)$ , numerically accurately, to at least around 12 decimal digits.

for tauJoe: in  $[-1, 1]$ .

## See Also

[acopula-families](#), and their class definition, "[acopula](#)". [etau\(\)](#) for method-of-moments estimators based on Kendall's tau.

## Examples

```
tauAMH(c(0, 2^-40, 2^-20))
curve(tauAMH, 0, 1)
curve(tauAMH, -1, 1)# negative taus as well
curve(tauAMH, 1e-12, 1, log="xy") # linear, tau ~ 2/9*theta in the limit

curve(tauJoe, 1, 10)
curve(tauJoe, 0.2387, 10)# negative taus (*not* valid for Joe: no 2-monotone psi(!))
```

---

uranium

*Uranium Exploration Dataset of Cook & Johnson (1986)*

---

## Description

These data consist of log concentrations of 7 chemical elements in 655 water samples collected near Grand Junction, CO (from the Montrose quad-range of Western Colorado). Concentrations were measured for the following elements: Uranium (U), Lithium (Li), Cobalt (Co), Potassium (K), Cesium (Cs), Scandium (Sc), And Titanium (Ti).

**Usage**

```
data(uranium, package="copula")
```

**Format**

A data frame with 655 observations of the following 7 variables:

U (numeric) log concentration of Uranium.  
 Li (numeric) log concentration of Lithium.  
 Co (numeric) log concentration of Colbalt.  
 K (numeric) log concentration of Potassium.  
 Cs (numeric) log concentration of Cesium.  
 Sc (numeric) log concentration of Scandum.  
 Ti (numeric) log concentration of Titanium.

**References**

Cook, R. D. and Johnson, M. E. (1986) Generalized BurrParetologistic distributions with applications to a uranium exploration data set. *Technometrics* **28**, 123–131.

**Examples**

```
data(uranium)
```

---

|                   |                                   |
|-------------------|-----------------------------------|
| varianceReduction | <i>Variance-Reduction Methods</i> |
|-------------------|-----------------------------------|

---

**Description**

Computing antithetic variates or Latin hypercube samples.

**Usage**

```
rAntitheticVariates(u)
rLatinHypercube(u, ...)
```

**Arguments**

`u` a  $n \times d$ -matrix (or  $d$ -vector) of random variates in the unit hypercube.  
`...` additional arguments passed to the underlying `rank()`.

**Details**

`rAntitheticVariates()` takes any copula sample  $u$ , builds  $1 - u$ , and returns the two matrices in the form of an array; this can be used for the variance-reduction method of (componentwise) antithetic variates.

`rLatinHypercube()` takes any copula sample, shifts its marginal ranks minus 1 by standard uniforms and then divides by the sample size in order to obtain a Latin hypercubed sample.



**Value**

rAntitheticVariates() **array** of dimension  $n \times d \times 2$ , say  $r$ , where  $r[, , 1]$  contains the original sample  $u$  and  $r[, , 2]$  contains the sample  $1-u$ .

rLatinHypercube() **matrix** of the same dimensions as  $u$ .

**References**

Cambou, M., Hofert, M. and Lemieux, C. (2016). Quasi-random numbers for copula models. *Statistics and Computing*, 1–23.

Packham, N. and Schmidt, W. M. (2010). Latin hypercube sampling with dependence and applications in finance. *Journal of Computational Finance* **13**(3), 81–111.

**Examples**

```
### 1 Basic plots #####

## Generate data from a Gumbel copula
cop <- gumbelCopula(iTau(gumbelCopula()), tau = 0.5)
n <- 1e4
set.seed(271)
U <- rCopula(n, copula = cop)

## Transform the sample to a Latin Hypercube sample
U.LH <- rLatinHypercube(U)

## Plot
## Note: The 'variance-reducing property' is barely visible, but that's okay
layout(rbind(1:2))
plot(U, xlab = quote(U[1]), ylab = quote(U[2]), pch = ".", main = "U")
plot(U.LH, xlab = quote(U[1]), ylab = quote(U[2]), pch = ".", main = "U.LH")
layout(1) # reset layout

## Transform the sample to an Antithetic variate sample
U.AV <- rAntitheticVariates(U)
stopifnot(identical(U.AV[,1], U),
          identical(U.AV[,2], 1-U))

## Plot original sample and its corresponding (componentwise) antithetic variates
layout(rbind(1:2))
plot(U.AV[,1], xlab = quote(U[1]), ylab = quote(U[2]), pch=".", main= "U")
plot(U.AV[,2], xlab = quote(U[1]), ylab = quote(U[2]), pch=".", main= "1 - U")
layout(1) # reset layout

### 2 Small variance-reduction study for exceedance probabilities #####

## Auxiliary function for approximately computing  $P(U_1 > u_1, \dots, U_d > u_d)$ 
## by Monte Carlo simulation based on pseudo-random numbers, Latin hypercube
## sampling and quasi-random numbers.
sProb <- function(n, copula, u)
{
```

```

d <- length(u)
stopifnot(n >= 1, inherits(copula, "Copula"), 0 < u, u < 1,
          d == dim(copula))
umat <- rep(u, each = n)
## Pseudo-random numbers
U <- rCopula(n, copula = copula)
PRNG <- mean(rowSums(U > umat) == d)
## Latin hypercube sampling (based on the recycled 'U')
U. <- rLatinHypercube(U)
LHS <- mean(rowSums(U. > umat) == d)
## Quasi-random numbers
U.. <- cCopula(sobol(n, d = d, randomize = TRUE), copula = copula,
              inverse = TRUE)
QRNG <- mean(rowSums(U.. > umat) == d)
## Return
c(PRNG = PRNG, LHS = LHS, QRNG = QRNG)
}

## Simulate the probabilities of falling in (u_1,1] x ... x (u_d,1]
library(qrng) # for quasi-random numbers
(Xtras <- copula::doExtras()) # determine whether examples will be extra (long)
B <- if(Xtras) 500 else 100 # number of replications
n <- if(Xtras) 1000 else 200 # sample size
d <- 2 # dimension; note: for d > 2, the true value depends on the seed
nu <- 3 # degrees of freedom
th <- iTau(tCopula(df = nu), tau = 0.5) # correlation parameter
cop <- tCopula(param = th, dim = d, df = nu) # t copula
u <- rep(0.99, d) # lower-left endpoint of the considered cube
set.seed(42) # for reproducibility
true <- prob(cop, l = u, u = rep(1, d)) # true exceedance probability
system.time(res <- replicate(B, sProb(n, copula = cop, u = u)))

## "abbreviations":
PRNG <- res["PRNG",]
LHS <- res["LHS" ,]
QRNG <- res["QRNG",]

## Compute the variance-reduction factors and % improvements
vrf <- var(PRNG) / var(LHS) # variance reduction factor w.r.t. LHS
vrf. <- var(PRNG) / var(QRNG) # variance reduction factor w.r.t. QRNG
pim <- (var(PRNG) - var(LHS)) / var(PRNG) *100 # improvement w.r.t. LHS
pim. <- (var(PRNG) - var(QRNG))/ var(PRNG) *100 # improvement w.r.t. QRNG

## Boxplot
boxplot(list(PRNG = PRNG, LHS = LHS, QRNG = QRNG), notch = TRUE,
        main = substitute("Simulated exceedance probabilities" ~
                          P(bold(U) > bold(u))~ "for a" ~ t[nu.]~"copula",
                          list(nu. = nu)),
        sub = sprintf(
          "Variance-reduction factors and %% improvements: %.1f (%%.0f%%), %.1f (%%.0f%%)",
          vrf, pim, vrf., pim.))
abline(h = true, lty = 3) # true value
mtext(sprintf("B = %d replications with n = %d and d = %d", B, n, d), side = 3)

```

---

 wireframe2-methods      *Perspective Plots via 'wireframe2'*


---

## Description

Generic function and methods `wireframe2()` to draw (**lattice**) **wireframe** (aka “perspective”) plots of two-dimensional distributions from package **copula**.

## Usage

```
## S4 method for signature 'matrix'
wireframe2(x,
  xlim = range(x[,1], finite = TRUE),
  ylim = range(x[,2], finite = TRUE),
  zlim = range(x[,3], finite = TRUE),
  xlab = NULL, ylab = NULL, zlab = NULL,
  alpha.regions = 0.5, scales = list(arrows = FALSE, col = "black"),
  par.settings = standard.theme(color = FALSE),
  draw.4.pCoplines = FALSE, ...)

## _identical_ method for 'data.frame' as for 'matrix'

## S4 method for signature 'Copula'
wireframe2(x, FUN, n.grid = 26, delta = 0,
  xlim = 0:1, ylim = 0:1, zlim = NULL,
  xlab = quote(u[1]), ylab = quote(u[2]),
  zlab = list(deparse(substitute(FUN)))[1], rot = 90),
  draw.4.pCoplines = identical(FUN, pCopula), ...)

## S4 method for signature 'mvdc'
wireframe2(x, FUN, n.grid = 26, xlim, ylim, zlim = NULL,
  xlab = quote(x[1]), ylab = quote(x[2]),
  zlab = list(deparse(substitute(FUN)))[1], rot = 90), ...)
```

## Arguments

|                               |                                                                                                                                                                                                                                                   |
|-------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <code>x</code>                | a "matrix", "data.frame", "Copula" or a "mvdc" object.                                                                                                                                                                                            |
| <code>xlim, ylim, zlim</code> | the x-, y- and z-axis limits.                                                                                                                                                                                                                     |
| <code>xlab, ylab, zlab</code> | the x-, y- and z-axis labels.                                                                                                                                                                                                                     |
| <code>scales</code>           | a <b>list</b> determining how the axes are drawn; see <code>wireframe()</code> .                                                                                                                                                                  |
| <code>FUN</code>              | the <b>function</b> to be plotted; for a "copula", typically <code>dCopula</code> or <code>pCopula</code> ; for an "mvdc", rather <code>dMvdc</code> , etc. In general of the form <code>function(x, copula)</code> .                             |
| <code>n.grid</code>           | the number of grid points used in each dimension. This can be a vector of length two, giving the number of grid points used in x- and y-direction, respectively; the function <code>FUN</code> will be evaluated on the corresponding (x,y)-grid. |

- `delta` a small number in  $[0, \frac{1}{2})$  influencing the evaluation boundaries. The x- and y-vectors will have the range  $[0+\text{delta}, 1-\text{delta}]$ , the default being  $[0, 1]$ .
- `draw.4.pCoplines` logical indicating if the 4 known border segments of a copula distribution function, i.e., `pCopula`, should be drawn. If true, the line segments are drawn with `col.4 = "#668b5580"`, `lwd.4 = 5`, and `lty.4 = "82"` which you can modify (via the ... below). Applies only when you do *not* set `panel.3d.wireframe` (via the ...).
- `alpha.regions, par.settings, ...` additional arguments passed to the underlying `wireframe()` from package **lattice**; includes `shade`, `drape`, `aspect`, etc., or (if you do not specify `panel.3d.wireframe` differently), to the package.

### Value

An object of class "trellis" as returned by `wireframe()`.

### Methods

Wireframe plots for objects of class "matrix", "data.frame", "Copula" or "mvdc".

### See Also

The [persp-methods](#) for drawing perspective plots via base graphics.

The **lattice**-based [contourplot2-methods](#).

### Examples

```
## For 'matrix' objects
## The Frechet--Hoeffding bounds W and M
n.grid <- 26
u <- seq(0, 1, length.out = n.grid)
grid <- expand.grid("u[1]" = u, "u[2]" = u)
W <- function(u) pmax(0, rowSums(u)-1) # lower bound W
M <- function(u) apply(u, 1, min) # upper bound M
x.W <- cbind(grid, "W(u[1],u[2])" = W(grid)) # evaluate W on 'grid'
x.M <- cbind(grid, "M(u[1],u[2])" = M(grid)) # evaluate M on 'grid'
wireframe2(x.W)
wireframe2(x.W, shade = TRUE) # plot of W
wireframe2(x.M, drape = TRUE) # plot of M
## use with custom colors:
myPalette <- colorRampPalette(c("white", "darkgreen", "gold", "red"))
myPar <- lattice::standard.theme(region = myPalette(16))
wireframe2(x.W, shade = TRUE, par.settings = myPar)

## For 'Copula' objects
cop <- frankCopula(-4)
wireframe2(cop, pCopula) # the copula
wireframe2(cop, pCopula, shade = TRUE) # ditto, "shaded" (but color=FALSE)
wireframe2(cop, pCopula, shade = TRUE, par.settings=list()) # use lattice default colors
wireframe2(cop, pCopula, shade = TRUE, col.4 = "gray60") # ditto, "shaded"+grid
```

```

wireframe2(cop, pCopula, drape = TRUE, xlab = quote(x[1])) # adjusting an axis label
wireframe2(cop, dCopula, delta=0.01) # the density
wireframe2(cop, dCopula, shade = TRUE, par.settings=list()) # use lattice default colors
wireframe2(cop, dCopula) # => the density is set to 0 on the margins
wireframe2(cop, function(u, copula) dCopula(u, copula, log=TRUE),
           shade = TRUE, par.settings = myPar,
           zlab = list(quote(log(c(u[1],u[2])))), rot=90, main = "dCopula(..., log=TRUE)")

## For 'mvdc' objects
mvNN <- mvdc(gumbelCopula(3), c("norm", "norm"),
            list(list(mean = 0, sd = 1), list(mean = 1)))
wireframe2(mvNN, dMvdc, xlim=c(-2, 2), ylim=c(-1, 3))
wireframe2(mvNN, dMvdc, xlim=c(-2, 2), ylim=c(-1, 3), shade=TRUE, par.settings=list())

```

xvCopula

*Model (copula) selection based on k-fold cross-validation*

## Description

Computes the leave-one-out cross-validation criterion (or a k-fold version of it) for the hypothesized parametric copula family using, by default, maximum pseudo-likelihood estimation.

The leave-one-out criterion is a crossvalidated log likelihood. It is denoted by  $\widehat{xv}_n$  in Grønneberg and Hjort (2014) and defined in equation (42) therein. When computed for several parametric copula families, it is thus meaningful to select the family maximizing the criterion.

For  $k < n$ ,  $n$  the sample size, the k-fold version is an approximation of the leave-one-out criterion that uses  $k$  randomly chosen (almost) equally sized data blocks instead of  $n$ . When  $n$  is large,  $k$ -fold cross-validation is considerably faster (if  $k$  is “small” compared to  $n$ ).

## Usage

```

xvCopula(copula, x, k = NULL, verbose = interactive(),
        ties.method = eval(formals(rank)$ties.method), ...)

```

## Arguments

|             |                                                                                                                                                                                              |
|-------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| copula      | object of class “ <a href="#">copula</a> ” representing the hypothesized copula family.                                                                                                      |
| x           | a data matrix that will be transformed to pseudo-observations.                                                                                                                               |
| k           | the number of data blocks; if $k = \text{NULL}$ , $\text{nrow}(x)$ blocks are considered (which corresponds to leave-one-out cross-validation).                                              |
| verbose     | a logical indicating if progress of the cross validation should be displayed via <a href="#">txtProgressBar</a> .                                                                            |
| ties.method | string specifying how ranks should be computed if there are ties in any of the coordinate samples of $x$ and fitting is based on maximum pseudo-likelihood; passed to <a href="#">pobs</a> . |
| ...         | additional arguments passed to <a href="#">fitCopula()</a> .                                                                                                                                 |

**Value**

A real number equal to the cross-validation criterion multiplied by the sample size.

**Note**

Note that k-fold cross-validation with  $k < n$  shuffles the lines of  $x$  prior to forming the blocks. The result thus depends on the value of the random seed.

The default estimation method is maximum pseudo-likelihood estimation but this can be changed if necessary along with all the other arguments of `fitCopula()`.

**References**

Grønneberg, S., and Hjort, N.L. (2014) The copula information criteria. *Scandinavian Journal of Statistics* **41**, 436–459.

**See Also**

`fitCopula()` for the underlying estimation procedure and `gofCopula()` for goodness-of-fit tests.

**Examples**

```
## A two-dimensional data example -----
x <- rCopula(200, claytonCopula(3))

## Model (copula) selection -- takes time: each fits 200 copulas to 199 obs.
xvCopula(gumbelCopula(), x)
xvCopula(frankCopula(), x)
xvCopula(joeCopula(), x)
xvCopula(claytonCopula(), x)
xvCopula(normalCopula(), x)
xvCopula(tCopula(), x)
xvCopula(plackettCopula(), x)

## The same with 5-fold cross-validation [to save time ...]
set.seed(1) # k-fold is random (for k < n) !
xvCopula(gumbelCopula(), x, k=5)
xvCopula(frankCopula(), x, k=5)
xvCopula(joeCopula(), x, k=5)
xvCopula(claytonCopula(), x, k=5)
xvCopula(normalCopula(), x, k=5)
xvCopula(tCopula(), x, k=5)
xvCopula(plackettCopula(), x, k=5)
```

# Index

- \* **Copula Family**
  - copFamilies, 35
- \* **Dilogarithm**
  - polylog, 168
- \* **Polylogarithm**
  - polylog, 168
- \* **arith**
  - Bernoulli, 22
  - coeffG, 30
  - interval, 119
  - polylog, 168
  - polynEval, 171
  - Stirling, 204
- \* **array**
  - matrix\_tools, 134
- \* **classes**
  - acopula-class, 10
  - archmCopula-class, 19
  - copula-class, 40
  - ellipCopula-class, 50
  - empCopula-class, 60
  - evCopula-class, 67
  - fgmCopula-class, 77
  - fhCopula-class, 79
  - fitCopula-class, 85
  - indepCopula-class, 113
  - interval-class, 120
  - khoudrajiCopula-class, 129
  - mixCopula-class, 138
  - moCopula-class, 140
  - mvdc-class, 147
  - nacopula-class, 149
  - plackettCopula-class, 163
- \* **datasets**
  - copFamilies, 35
  - gasoil, 92
  - loss, 131
  - rdj, 177
  - SMI.12, 201
  - uranium, 207
- \* **dilog**
  - polylog, 168
- \* **distribution**
  - absdPsiMC, 8
  - acR, 13
  - archmCopula, 17
  - beta.Blomqvist, 23
  - cCopula, 25
  - Copula, 38
  - dDiag, 43
  - dnacopula, 45
  - ellipCopula, 46
  - evCopula, 65
  - fgmCopula, 75
  - fhCopula, 78
  - gnacopula, 99
  - gofOtherTstat, 108
  - gofTstat, 109
  - htrafo, 111
  - indepCopula, 113
  - K, 121
  - khoudrajiCopula, 125
  - mixCopula, 136
  - Mvdc, 145
  - opower, 154
  - plackettCopula, 162
  - pnacopula, 165
  - prob, 173
  - retstable, 178
  - rF01FrankJoe, 179
  - rFFrankJoe, 181
  - rlog, 182
  - rnacModel, 183
  - rnacopula, 184
  - rnchild, 185
  - rotCopula, 187
  - rstable1, 191
  - Sibuya, 199

- tauAMH, 206
- varianceReduction, 208
- \* **goodness-of-fit**
  - gofCopula, 101
  - gofOtherTstat, 108
  - gofTstat, 109
- \* **hplot**
  - .pairsCond, 7
  - cloud2-methods, 29
  - contour-methods, 31
  - contourplot2-methods, 33
  - pairs2, 156
  - pairsRosenblatt, 157
  - persp-methods, 161
  - plot-methods, 164
  - qqplot2, 174
  - splom2-methods, 203
  - wireframe2-methods, 211
- \* **htest**
  - An, 15
  - evTestA, 68
  - evTestC, 70
  - evTestK, 71
  - exchEVTest, 72
  - exchTest, 74
  - ggraph-tools, 98
  - gnacopula, 99
  - gofCopula, 101
  - gofEVCopula, 106
  - gofOtherTstat, 108
  - gofTstat, 109
  - indepTest, 114
  - multIndepTest, 141
  - multSerialIndepTest, 143
  - radSymTest, 175
  - serialIndepTest, 195
- \* **manip**
  - allComp, 14
  - fixParam, 91
  - getAcop, 94
  - getTheta, 97
  - matrix\_tools, 134
  - setTheta, 197
- \* **math**
  - log1mexp, 130
  - math-fun, 133
- \* **methods**
  - cloud2-methods, 29
  - contour-methods, 31
  - contourplot2-methods, 33
  - describeCop, 44
  - getTheta, 97
  - persp-methods, 161
  - prob, 173
  - show-methods, 199
  - varianceReduction, 208
  - wireframe2-methods, 211
- \* **models**
  - emde, 51
  - emle, 53
  - enacopula, 60
  - estim.misc, 63
  - fitCopula, 79
  - fitMvdc, 88
  - getIniParam, 96
  - gofCopula, 101
  - gofEVCopula, 106
  - margCopula, 132
  - xvCopula, 213
- \* **multivariate**
  - .pairsCond, 7
  - acopula-class, 10
  - An, 15
  - archmCopula, 17
  - assocMeasures, 21
  - beta.Blomqvist, 23
  - Copula, 38
  - ellipCopula, 46
  - empCopula, 56
  - evCopula, 65
  - evTestA, 68
  - evTestC, 70
  - evTestK, 71
  - exchEVTest, 72
  - exchTest, 74
  - fgmCopula, 75
  - fhCopula, 78
  - fitCopula, 79
  - fitLambda, 86
  - fitMvdc, 88
  - generator, 93
  - ggraph-tools, 98
  - gnacopula, 99
  - gofCopula, 101
  - gofEVCopula, 106
  - gofOtherTstat, 108



- gofTstat, 109
- htrafo, 111
- indepCopula, 113
- khoudrajiCopula, 125
- margCopula, 132
- mixCopula, 136
- moCopula, 139
- Mvdc, 145
- nacopula-class, 149
- onacopula, 152
- pairsRosenblatt, 157
- plackettCopula, 162
- pnacopula, 165
- radSymTest, 175
- rnacModel, 183
- rotCopula, 187
- xvCopula, 213
- \* **nonparametric**
  - fitLambda, 86
- \* **optimize**
  - safeUroot, 193
- \* **package**
  - copula-package, 5
- \* **print**
  - show-methods, 199
- \* **reflected copula**
  - rotCopula, 187
- \* **rotated copula**
  - rotCopula, 187
- \* **survival copula**
  - rotCopula, 187
- \* **transformation**
  - htrafo, 111
- \* **utilities**
  - allComp, 14
  - describeCop, 44
  - fixParam, 91
  - interval, 119
  - nacFrail.time, 148
  - nacPairthetas, 150
  - nesdepth, 151
  - printNacopula, 172
  - RSpobs, 189
- .ac.classNameNames (getAcop), 94
- .ac.longNames (getAcop), 94
- .ac.objNames (getAcop), 94
- .ac.shortNames (getAcop), 94
- .emle, 61
- .emle (emle), 53
- .pairsCond, 7, 158, 159
- %in%, numeric, interval-method
  - (interval-class), 120
- A, 16
- A (generator), 93
- A, galambosCopula-method (generator), 93
- A, gumbelCopula-method (generator), 93
- A, huslerReissCopula-method (generator), 93
- A, indepCopula-method (generator), 93
- A, khoudrajiCopula-method (generator), 93
- A, tawnCopula-method (generator), 93
- A, tevCopula-method (generator), 93
- A-methods (generator), 93
- A..Z (math-fun), 133
- absdPsiMC, 8
- acopula, 5, 18, 21, 24, 35, 36, 39, 41, 43, 45, 46, 95, 121, 148, 149, 152–155, 197, 207
- acopula (acopula-class), 10
- acopula-class, 10
- acopula-families, 94
- acopula-families (copFamilies), 35
- acR, 13
- Afun (generator), 93
- AfunDer (generator), 93
- algorithms, 47
- allComp, 14
- amhCopula (archmCopula), 17
- amhCopula-class (archmCopula-class), 19
- An, 15, 66, 69, 70, 72, 94, 107
- Anfun (An), 15
- archmCopula, 5, 17, 19, 20, 39, 41, 48, 66–68, 79, 95, 111, 113, 146, 163
- archmCopula-class, 19
- array, 98, 99, 157, 209
- as.matrix, 156
- assocMeasures, 21
- asym2Copula-class
  - (khoudrajiCopula-class), 129
- asymCopula (khoudrajiCopula), 125
- asymCopula-class
  - (khoudrajiCopula-class), 129
- asymExplicitCopula (khoudrajiCopula), 125
- attr, 48
- attributes, 48, 97

- axis, 7
- Bernoulli, 22
- beta. (beta.Blomqvist), 23
- beta.Blomqvist, 23
- beta.hat (beta.Blomqvist), 23
- betan (beta.Blomqvist), 23
- C.n (empCopula), 56
- cacopula (cCopula), 25
- calibKendallsTau (assocMeasures), 21
- calibSpearmanRho (assocMeasures), 21
- cCopula, 25, 100, 101, 103
- character, 15, 31, 44, 47, 48, 51, 52, 56, 60, 61, 68, 80, 95, 99–102, 109, 117, 135, 140, 145, 152, 156, 157, 163, 167, 172, 174, 187, 189, 198, 203
- class, 69–71, 73, 75, 104, 107, 120, 136, 147, 176, 184
- claytonCopula (archmCopula), 17
- claytonCopula-class (archmCopula-class), 19
- cloud, 29, 30
- cloud2 (cloud2-methods), 29
- cloud2, Copula-method (cloud2-methods), 29
- cloud2, data.frame-method (cloud2-methods), 29
- cloud2, matrix-method (cloud2-methods), 29
- cloud2, mvdc-method (cloud2-methods), 29
- cloud2-methods, 29
- Cn (empCopula), 56
- coef, 82, 86, 89
- coef.fittedMV (fitMvdc), 88
- coeffG, 30
- complex\_dilog, 168
- contour, 31, 32
- contour, Copula-method (contour-methods), 31
- contour, indepCopula-method (contour-methods), 31
- contour, mvdc-method (contour-methods), 31
- contour-methods, 31
- contourplot, 33, 34
- contourplot2 (contourplot2-methods), 33
- contourplot2, Copula-method (contourplot2-methods), 33
- contourplot2, data.frame-method (contourplot2-methods), 33
- contourplot2, matrix-method (contourplot2-methods), 33
- contourplot2, mvdc-method (contourplot2-methods), 33
- contourplot2-methods, 33
- copAMH, 12, 18
- copAMH (copFamilies), 35
- copClayton, 12, 17, 18, 38
- copClayton (copFamilies), 35
- copFamilies, 35
- copFrank, 12, 182
- copFrank (copFamilies), 35
- copGumbel, 6, 12, 18, 21, 38, 153, 185
- copGumbel (copFamilies), 35
- copJoe, 12
- copJoe (copFamilies), 35
- Copula, 26, 29, 30, 32–34, 38, 38, 47, 76, 83, 90, 138, 161, 164, 173, 187, 198, 199, 203, 211, 212
- copula, 20, 21, 38, 39, 44, 51, 67, 68, 77, 80, 86, 91–93, 96, 97, 102, 126, 129, 132, 141, 145–147, 162, 163, 187, 213
- Copula-class (copula-class), 40
- copula-class, 40
- copula-package, 5
- cor, 42, 64
- cor.fk, 41, 42, 64, 117
- corKendall, 41, 63, 64
- dAdu (generator), 93
- dAdu, galambosCopula-method (generator), 93
- dAdu, gumbelCopula-method (generator), 93
- dAdu, huslerReissCopula-method (generator), 93
- dAdu, tawnCopula-method (generator), 93
- dAdu, tevCopula-method (generator), 93
- dAdu-methods (generator), 93
- data.frame, 29, 30, 33, 34, 135, 142, 164, 203, 211, 212
- Date, 92, 177
- dCn (empCopula), 56
- dCopula, 5, 18, 20, 32, 34, 45, 51, 78, 80, 129, 161, 211
- dCopula (Copula), 38
- dcopula (Copula), 38

- dCopula, matrix, amhCopula-method (Copula), 38
- dCopula, matrix, claytonCopula-method (Copula), 38
- dCopula, matrix, empCopula-method (Copula), 38
- dCopula, matrix, fgmCopula-method (Copula), 38
- dCopula, matrix, fhCopula-method (Copula), 38
- dCopula, matrix, frankCopula-method (Copula), 38
- dCopula, matrix, galambosCopula-method (Copula), 38
- dCopula, matrix, gumbelCopula-method (Copula), 38
- dCopula, matrix, huslerReissCopula-method (Copula), 38
- dCopula, matrix, indepCopula-method (Copula), 38
- dCopula, matrix, joeCopula-method (Copula), 38
- dCopula, matrix, khoudrajiBivCopula-method (Copula), 38
- dCopula, matrix, khoudrajiExplicitCopula-method (Copula), 38
- dCopula, matrix, mixCopula-method (Copula), 38
- dCopula, matrix, moCopula-method (Copula), 38
- dCopula, matrix, nacopula-method (dnacopula), 45
- dCopula, matrix, normalCopula-method (Copula), 38
- dCopula, matrix, plackettCopula-method (Copula), 38
- dCopula, matrix, rotCopula-method (Copula), 38
- dCopula, matrix, rotExplicitCopula-method (Copula), 38
- dCopula, matrix, tawnCopula-method (Copula), 38
- dCopula, matrix, tCopula-method (Copula), 38
- dCopula, matrix, tevCopula-method (Copula), 38
- dCopula, numeric, nacopula-method (dnacopula), 45
- dDiag, 43
- debye1 (polylog), 168
- debye2 (polylog), 168
- debye\_1, 168
- demo, 55, 62, 99, 149, 159
- dependogram, 142, 144, 196
- dependogram (indepTest), 114
- describeCop, 44
- describeCop, archmCopula, character-method (describeCop), 44
- describeCop, copula, character-method (describeCop), 44
- describeCop, Copula, missing-method (describeCop), 44
- describeCop, ellipCopula, character-method (describeCop), 44
- describeCop, empCopula, character-method (describeCop), 44
- describeCop, fgmCopula, character-method (describeCop), 44
- describeCop, fhCopula, character-method (describeCop), 44
- describeCop, indepCopula, character-method (describeCop), 44
- describeCop, khoudrajiCopula, character-method (describeCop), 44
- describeCop, mixCopula, character-method (describeCop), 44
- describeCop, moCopula, character-method (describeCop), 44
- describeCop, rotCopula, character-method (describeCop), 44
- describeCop, Xcopula, ANY-method (describeCop), 44
- describeCop-methods (describeCop), 44
- digamma, 207
- dilog, 168
- dim, 14, 40, 150, 152
- dim, copula-method (copula-class), 40
- dim, dimCopula-method (copula-class), 40
- dim, empCopula-method (empCopula-class), 60
- dim, khoudrajiCopula-method (khoudrajiCopula-class), 129
- dim, mixCopula-method (mixCopula-class), 138
- dim, mvdc-method (mvdc-class), 147
- dim, nacopula-method (nacopula-class),

- 149
- dim, rotCopula-method (rotCopula), 187
- dim, Xcopula-method (copula-class), 40
- dimCopula, 79, 114
- dimCopula-class (copula-class), 40
- diPsi (generator), 93
- diPsi, amhCopula-method (generator), 93
- diPsi, claytonCopula-method (generator), 93
- diPsi, frankCopula-method (generator), 93
- diPsi, gumbelCopula-method (generator), 93
- diPsi, joeCopula-method (generator), 93
- diPsi-methods (generator), 93
- dispstrToep (ellipCopula), 46
- dK (K), 121
- dMvdc, 211
- dMvdc (Mvdc), 145
- dmvdc (Mvdc), 145
- dmvt, 38
- dnacopula, 43, 45
- dnorm, 146
- double, 200
- dSibuya (Sibuya), 199
- dsumSibuya, 31
- dsumSibuya (Sibuya), 199
- ebeta, 62, 118
- ebeta (estim.misc), 63
- edmle, 61, 62, 118
- edmle (estim.misc), 63
- ellipCopula, 5, 18, 39, 41, 46, 47, 50, 51, 66, 79, 82, 104, 113, 134, 135, 146, 163
- ellipCopula-class, 50
- emde, 51, 61, 62, 64, 118, 123
- emle, 53, 62, 64, 83, 118, 170
- empCopula, 56, 57, 60
- empCopula-class, 60
- enacopula, 53–55, 60, 62, 64, 100, 118
- environment, 154
- estim.misc, 63
- etau, 42, 62, 207
- etau (estim.misc), 63
- Eulerian, 6, 23, 169
- Eulerian (Stirling), 204
- evCopula, 5, 16, 18, 20, 40, 41, 65, 68–70, 72, 79, 106, 107, 113
- evCopula-class, 67
- evTestA, 16, 68, 70, 72, 107
- evTestC, 16, 68, 69, 70, 72, 107
- evTestK, 16, 68–70, 71, 107
- exchEVTTest, 16, 72, 75, 176
- exchTest, 73, 74, 176
- expression, 79, 114, 129, 140, 163
- extremePairs (matrix\_tools), 134
- F.n (empCopula), 56
- FALSE, 52
- fgmCopula, 39, 41, 75, 76–78
- fgmCopula-class, 77
- fhCopula, 78, 79
- fhCopula-class, 79
- findInterval, 122
- fitCopula, 5, 18, 42, 47, 48, 66, 76, 79, 82, 85, 88, 90–92, 96, 102, 103, 105, 107, 139, 146, 188, 199, 213, 214
- fitCopula, copula-method (fitCopula), 79
- fitCopula, parCopula-method (fitCopula), 79
- fitCopula, rotCopula-method (fitCopula), 79
- fitCopula-class, 85
- fitCopula-methods (fitCopula), 79
- fitdistr, 90
- fitLambda, 86
- fitMvdc, 83, 85, 86, 88, 89, 90, 145, 146, 148, 199
- fitMvdc-class (fitCopula-class), 85
- fittedMV-class (fitCopula-class), 85
- fixedParam<- (fixParam), 91
- fixedParam<- , copula, logical-method (fixParam), 91
- fixedParam<- , khoudrajiCopula, logical-method (fixParam), 91
- fixedParam<- , mixCopula, logical-method (fixParam), 91
- fixedParam<- , rotCopula, logical-method (fixParam), 91
- fixParam, 91, 139, 198
- formals, 146
- format, interval-method (interval-class), 120
- frankCopula (archmCopula), 17
- frankCopula-class (archmCopula-class), 19
- function, 10, 11, 32, 34, 81, 99, 157, 158, 161, 183, 189, 211

- galambosCopula, [66](#), [67](#)
- galambosCopula (evCopula), [65](#)
- galambosCopula-class (evCopula-class), [67](#)
- gasoil, [92](#)
- generator, [93](#)
- genFun (generator), [93](#)
- genFunDer1 (generator), [93](#)
- genFunDer2 (generator), [93](#)
- genInv (generator), [93](#)
- GenzBretz, [48](#)
- getAcop, [12](#), [36](#), [94](#)
- getAname (getAcop), [94](#)
- getIniParam, [96](#)
- getIniParam, mixCopula-method (getIniParam), [96](#)
- getIniParam, parCopula-method (getIniParam), [96](#)
- getSigma, [47](#), [48](#)
- getSigma (matrix\_tools), [134](#)
- getTheta, [80](#), [96](#), [97](#), [139](#), [198](#)
- getTheta, acopula-method (getTheta), [97](#)
- getTheta, copula-method (getTheta), [97](#)
- getTheta, khoudrajiCopula-method (getTheta), [97](#)
- getTheta, mixCopula-method (getTheta), [97](#)
- getTheta, parCopula-method (getTheta), [97](#)
- getTheta, rotCopula-method (getTheta), [97](#)
- getTheta, Xcopula-method (getTheta), [97](#)
- getTheta-methods (getTheta), [97](#)
- ggraph-tools, [98](#)
- gnacopula, [99](#)
- gofBTstat (gofOtherTstat), [108](#)
- gofCopula, [53](#), [73](#), [75](#), [83](#), [90](#), [100](#), [101](#), [101](#), [107](#), [109](#), [111](#), [176](#), [188](#), [197](#), [214](#)
- gofCopula, copula-method (gofCopula), [101](#)
- gofCopula, parCopula-method (gofCopula), [101](#)
- gofCopula, rotCopula-method (gofCopula), [101](#)
- gofCopula-methods (gofCopula), [101](#)
- gofEVCopula, [16](#), [66](#), [68–70](#), [72](#), [106](#)
- gofMB (gofCopula), [101](#)
- gofOtherTstat, [108](#)
- gofPB (gofCopula), [101](#)
- gofT2stat (gofTstat), [109](#)
- gofTstat, [100](#), [101](#), [105](#), [109](#)
- gpviTest (ggraph-tools), [98](#)
- grad, [81](#), [103](#)
- gumbelCopula, [66](#), [68](#)
- gumbelCopula (archmCopula), [17](#)
- gumbelCopula-class (archmCopula-class), [19](#)
- htrafo, [27](#), [52](#), [53](#), [100](#), [101](#), [103](#), [111](#), [123](#)
- huslerReissCopula, [66](#)
- huslerReissCopula (evCopula), [65](#)
- huslerReissCopula-class (evCopula-class), [67](#)
- image, [7](#)
- indepCopula, [17](#), [113](#), [113](#), [114](#)
- indepCopula-class, [113](#)
- indepTest, [98](#), [99](#), [114](#), [141](#), [142](#), [144](#), [195](#), [196](#)
- indepTestSim, [98](#)
- indepTestSim (indepTest), [114](#)
- initialize, acopula-method (acopula-class), [10](#)
- initOpt, [117](#)
- integer, [14](#), [48](#), [54](#), [56](#), [61](#), [98](#), [121](#), [149](#), [180](#), [182](#), [186](#), [187](#), [192](#), [200](#), [201](#)
- interval, [11](#), [12](#), [119](#), [119](#), [120](#)
- interval-class, [120](#)
- invisible, [156](#), [161](#), [175](#)
- iPsi (generator), [93](#)
- iPsi, amhCopula-method (generator), [93](#)
- iPsi, claytonCopula-method (generator), [93](#)
- iPsi, frankCopula-method (generator), [93](#)
- iPsi, gumbelCopula-method (generator), [93](#)
- iPsi, joeCopula-method (generator), [93](#)
- iPsi-methods (generator), [93](#)
- iRho (assocMeasures), [21](#)
- iRho, ANY-method (assocMeasures), [21](#)
- iRho, archmCopula-method (assocMeasures), [21](#)
- iRho, claytonCopula-method (assocMeasures), [21](#)
- iRho, copula-method (assocMeasures), [21](#)
- iRho, ellipCopula-method (assocMeasures), [21](#)
- iRho, fgmCopula-method (assocMeasures), [21](#)
- iRho, frankCopula-method (assocMeasures), [21](#)

- iRho, galambosCopula-method (assocMeasures), 21
- iRho, gumbelCopula-method (assocMeasures), 21
- iRho, huslerReissCopula-method (assocMeasures), 21
- iRho, nacopula-method (assocMeasures), 21
- iRho, normalCopula-method (assocMeasures), 21
- iRho, plackettCopula-method (assocMeasures), 21
- iRho, rotCopula-method (assocMeasures), 21
- iRho, tawnCopula-method (assocMeasures), 21
- iRho, tCopula-method (assocMeasures), 21
- iRho, tevCopula-method (assocMeasures), 21
- iRho-methods (assocMeasures), 21
- isFree, 80
- isFree (fixParam), 91
- isFree, copula-method (fixParam), 91
- isFree, khoudrajiCopula-method (fixParam), 91
- isFree, mixCopula-method (fixParam), 91
- isFree, parCopula-method (fixParam), 91
- isFree, rotCopula-method (fixParam), 91
- isFreeP (fixParam), 91
- iTau, 96
- iTau (assocMeasures), 21
- iTau, acopula-method (assocMeasures), 21
- iTau, amhCopula-method (assocMeasures), 21
- iTau, ANY-method (assocMeasures), 21
- iTau, archmCopula-method (assocMeasures), 21
- iTau, claytonCopula-method (assocMeasures), 21
- iTau, copula-method (assocMeasures), 21
- iTau, ellipCopula-method (assocMeasures), 21
- iTau, fgmCopula-method (assocMeasures), 21
- iTau, frankCopula-method (assocMeasures), 21
- iTau, galambosCopula-method (assocMeasures), 21
- iTau, gumbelCopula-method (assocMeasures), 21
- iTau, huslerReissCopula-method (assocMeasures), 21
- iTau, joeCopula-method (assocMeasures), 21
- iTau, nacopula-method (assocMeasures), 21
- iTau, normalCopula-method (assocMeasures), 21
- iTau, plackettCopula-method (assocMeasures), 21
- iTau, rotCopula-method (assocMeasures), 21
- iTau, tawnCopula-method (assocMeasures), 21
- iTau, tCopula-method (assocMeasures), 21
- iTau, tevCopula-method (assocMeasures), 21
- iTau-methods (assocMeasures), 21
- joeCopula (archmCopula), 17
- joeCopula-class (archmCopula-class), 19
- K, 53, 100, 101, 111, 121, 123
- kendallsTau (assocMeasures), 21
- khoudrajiBivCopula, 125, 126
- khoudrajiBivCopula-class (khoudrajiCopula-class), 129
- khoudrajiCopula, 125, 125, 126, 129, 130, 136
- khoudrajiCopula-class, 129
- khoudrajiExplicitCopula, 125, 126
- khoudrajiExplicitCopula-class (khoudrajiCopula-class), 129
- Kn (K), 121
- lambda, 87, 136, 139
- lambda (assocMeasures), 21
- lambda, acopula-method (assocMeasures), 21
- lambda, amhCopula-method (assocMeasures), 21
- lambda, ANY-method (assocMeasures), 21
- lambda, claytonCopula-method (assocMeasures), 21
- lambda, copula-method (assocMeasures), 21
- lambda, evCopula-method (assocMeasures), 21
- lambda, frankCopula-method (assocMeasures), 21

- lambda, gumbelCopula-method (assocMeasures), 21
- lambda, indepCopula-method (assocMeasures), 21
- lambda, joeCopula-method (assocMeasures), 21
- lambda, lowfhCopula-method (assocMeasures), 21
- lambda, mixCopula-method (mixCopula-class), 138
- lambda, moCopula-method (assocMeasures), 21
- lambda, nacopula-method (assocMeasures), 21
- lambda, normalCopula-method (assocMeasures), 21
- lambda, plackettCopula-method (assocMeasures), 21
- lambda, rotCopula-method (assocMeasures), 21
- lambda, tCopula-method (assocMeasures), 21
- lambda, upfhCopula-method (assocMeasures), 21
- lambda-methods (assocMeasures), 21
- lines, 174, 175
- list, 8, 29, 34, 52, 54, 64, 80, 81, 86, 87, 103, 136, 138, 145, 146, 149, 152, 153, 157–159, 174, 190, 211
- log, 31, 43, 45, 93, 146
- log1mexp, 130
- log1pexp (log1mexp), 130
- logical, 13, 26, 34, 48, 68, 81, 87, 89, 92, 96, 98, 117, 120, 121, 135, 167, 187, 189, 200
- logLik, 82, 86
- logLik.fittedMV (fitMvdc), 88
- loglikCopula, 96
- loglikCopula (fitCopula), 79
- loglikCopulaMany (fitCopula), 79
- loglikMvdc (fitMvdc), 88
- loss, 131
- lowfhCopula, 78
- lowfhCopula (fhCopula), 78
- lowfhCopula-class (fhCopula-class), 79
- lSMI (SMI.12), 201
- margCopula, 132
- margCopula, archmCopula, logical-method (margCopula), 132
- margCopula, normalCopula, logical-method (margCopula), 132
- margCopula, tCopula, logical-method (margCopula), 132
- math-fun, 133
- matrix, 26, 29, 30, 33, 34, 56, 60, 87, 89, 98, 99, 108, 134, 135, 156–159, 164, 167, 189, 190, 202, 203, 208, 209, 211, 212
- matrix\_tools, 134
- maybeInterval-class (interval-class), 120
- methods, 51
- min, 120
- missing, 42
- mixCopula, 86, 89, 136, 136, 138, 139
- mixCopula-class, 138
- mle, 54, 55, 62
- mle2, 54, 55
- moCopula, 139, 139, 140, 141
- moCopula-class, 140
- mpfr, 31, 200
- mtext, 174, 175
- multIndepTest, 116, 141, 144, 196
- multSerialIndepTest, 116, 142, 143, 196
- Mvdc, 145
- mvdc, 29, 30, 32–34, 86, 88–90, 146–148, 161, 162, 164, 203, 211, 212
- mvdc (Mvdc), 145
- mvdc-class, 147
- NA, 38, 41, 42, 64, 169, 198
- NA\_real\_, 40
- nac2list (onacopula), 152
- nacFrail.time, 148
- nacopula, 12, 14, 21, 38, 41, 45, 150–153, 165, 172, 184–186
- nacopula (onacopula), 152
- nacopula-class, 149
- nacPairthetas, 150
- names, 96, 97
- NaN, 38
- nearPD, 82
- nesdepth, 151
- nobs, 82
- normalCopula, 48, 50, 51, 134, 135
- normalCopula (ellipCopula), 46

- normalCopula-class (ellipCopula-class), 50
- nParam (fixParam), 91
- nParam, copula-method (fixParam), 91
- nParam, khoudrajiCopula-method (fixParam), 91
- nParam, mixCopula-method (fixParam), 91
- nParam, parCopula-method (fixParam), 91
- nParam, rotCopula-method (fixParam), 91
- NULL, 11, 193
- numeric, 9, 10, 17, 40, 43, 45, 47, 51, 57, 60, 92, 96, 97, 110, 120, 122, 126, 133, 138, 139, 148, 158, 165, 166, 173, 174, 185–187, 192, 198, 202
- onacopula, 12, 36, 149, 152, 165, 184–186, 197
- onacopulaL (onacopula), 152
- opower, 154
- optim, 54, 81, 82, 86, 89, 90, 107
- optimize, 52, 54, 61, 63, 64, 81, 87
- optimMeth (fitCopula), 79
- outer\_nacopula, 14, 43, 45, 52, 54, 61, 63, 100, 111, 121, 152, 153, 183–186, 197
- outer\_nacopula-class (nacopula-class), 149
- p.adjust.methods, 99
- P2p (matrix\_tools), 134
- p2P, 47, 48
- p2P (matrix\_tools), 134
- pacR (acR), 13
- pairs, 7, 156, 158, 204
- pairs2, 156, 204
- pairsCollist, 7
- pairsCollist (pairsRosenblatt), 157
- pairsRosenblatt, 7, 8, 99, 157
- pairwiseCcop, 7, 157, 159
- pairwiseCcop (ggraph-tools), 98
- pairwiseIndepTest (ggraph-tools), 98
- par, 7
- parCopula, 44, 96, 97, 114, 136, 138
- parCopula-class (copula-class), 40
- pCopula, 5, 20, 32, 34, 48, 51, 161, 166, 173, 211, 212
- pCopula (Copula), 38
- pcopula (Copula), 38
- pCopula, matrix, amhCopula-method (Copula), 38
- pCopula, matrix, claytonCopula-method (Copula), 38
- pCopula, matrix, empCopula-method (Copula), 38
- pCopula, matrix, fgmCopula-method (Copula), 38
- pCopula, matrix, frankCopula-method (Copula), 38
- pCopula, matrix, galambosCopula-method (Copula), 38
- pCopula, matrix, gumbelCopula-method (Copula), 38
- pCopula, matrix, huslerReissCopula-method (Copula), 38
- pCopula, matrix, indepCopula-method (Copula), 38
- pCopula, matrix, joeCopula-method (Copula), 38
- pCopula, matrix, khoudrajiCopula-method (Copula), 38
- pCopula, matrix, lowfhCopula-method (Copula), 38
- pCopula, matrix, mixCopula-method (Copula), 38
- pCopula, matrix, moCopula-method (Copula), 38
- pCopula, matrix, nacopula-method (pnacopula), 165
- pCopula, matrix, normalCopula-method (ellipCopula), 46
- pCopula, matrix, plackettCopula-method (Copula), 38
- pCopula, matrix, rotCopula-method (Copula), 38
- pCopula, matrix, rotExplicitCopula-method (Copula), 38
- pCopula, matrix, tawnCopula-method (Copula), 38
- pCopula, matrix, tCopula-method (ellipCopula), 46
- pCopula, matrix, tevCopula-method (Copula), 38
- pCopula, matrix, upfhCopula-method (Copula), 38
- pCopula, numeric, nacopula-method (pnacopula), 165



- pdf, [175](#)
- persp, [161](#)
- persp, Copula-method (persp-methods), [161](#)
- persp, mvdc-method (persp-methods), [161](#)
- persp-methods, [161](#)
- persp.default, [161](#)
- pK (K), [121](#)
- plackettCopula, [162](#), [162](#), [163](#), [164](#)
- plackettCopula-class, [163](#)
- plot, [44](#), [164](#), [175](#)
- plot, Copula, ANY-method (plot-methods), [164](#)
- plot, mvdc, ANY-method (plot-methods), [164](#)
- plot-methods, [164](#)
- plot.default, [164](#)
- pMvdc (Mvdc), [145](#)
- pmvdc (Mvdc), [145](#)
- pmvnorm, [48](#)
- pmvt, [48](#)
- pnacopula, [93](#), [165](#)
- pobs, [15](#), [56](#), [58](#), [61](#), [68](#), [73](#), [74](#), [80](#), [82](#), [88](#), [100](#), [102](#), [103](#), [106](#), [111](#), [146](#), [166](#), [189](#), [190](#), [213](#)
- points, [158](#)
- polylog, [6](#), [168](#)
- polyn.eval, [171](#)
- polynEval, [171](#)
- predict.polynomial, [171](#)
- print, [44](#), [199](#)
- print.default, [172](#)
- printNacopula, [149](#), [172](#)
- prob, [6](#), [173](#)
- prob, Copula-method (prob), [173](#)
- prob-methods (prob), [173](#)
- psi (generator), [93](#)
- psi, amhCopula-method (generator), [93](#)
- psi, claytonCopula-method (generator), [93](#)
- psi, frankCopula-method (generator), [93](#)
- psi, gumbelCopula-method (generator), [93](#)
- psi, joeCopula-method (generator), [93](#)
- psi-methods (generator), [93](#)
- pSibuya (Sibuya), [199](#)
- psiDabsMC (absdPsiMC), [8](#)
- psigamma, [206](#)
- pviTest (ggraph-tools), [98](#)
- qacR (acR), [13](#)
- qK, [111](#)
- qK (K), [121](#)
- qqline, [158](#), [174](#), [175](#)
- qqplot2, [174](#)
- radSymTest, [73](#), [75](#), [175](#)
- range, [32](#), [120](#), [161](#)
- rank, [60](#), [167](#), [208](#)
- rAntitheticVariates (varianceReduction), [208](#)
- rCopula, [5](#), [51](#)
- rCopula (Copula), [38](#)
- rcopula (Copula), [38](#)
- rCopula, numeric, amhCopula-method (Copula), [38](#)
- rCopula, numeric, claytonCopula-method (Copula), [38](#)
- rCopula, numeric, empCopula-method (Copula), [38](#)
- rCopula, numeric, evCopula-method (Copula), [38](#)
- rCopula, numeric, fgmCopula-method (Copula), [38](#)
- rCopula, numeric, frankCopula-method (Copula), [38](#)
- rCopula, numeric, galambosCopula-method (Copula), [38](#)
- rCopula, numeric, gumbelCopula-method (Copula), [38](#)
- rCopula, numeric, huslerReissCopula-method (Copula), [38](#)
- rCopula, numeric, indepCopula-method (Copula), [38](#)
- rCopula, numeric, joeCopula-method (Copula), [38](#)
- rCopula, numeric, khoudrajiCopula-method (Copula), [38](#)
- rCopula, numeric, lowfhCopula-method (Copula), [38](#)
- rCopula, numeric, mixCopula-method (Copula), [38](#)
- rCopula, numeric, moCopula-method (Copula), [38](#)
- rCopula, numeric, nacopula-method (Copula), [38](#)
- rCopula, numeric, normalCopula-method (Copula), [38](#)
- rCopula, numeric, plackettCopula-method (Copula), [38](#)
- rCopula, numeric, rotCopula-method (Copula), [38](#)

- rCopula, numeric, tCopula-method (Copula), [38](#)
- rCopula, numeric, upfhCopula-method (Copula), [38](#)
- rdj, [177](#)
- retstable, [134](#), [178](#)
- retstableR (retstable), [178](#)
- rF01Frank, [181](#)
- rF01Frank (rF01FrankJoe), [179](#)
- rF01FrankJoe, [179](#)
- rF01Joe, [181](#), [201](#)
- rF01Joe (rF01FrankJoe), [179](#)
- rFFrank, [180](#)
- rFFrank (rFFrankJoe), [181](#)
- rFFrankJoe, [181](#)
- rFJoe, [180](#), [201](#)
- rFJoe (rFFrankJoe), [181](#)
- rho, [5](#), [136](#), [170](#)
- rho (assocMeasures), [21](#)
- rho, acopula-method (assocMeasures), [21](#)
- rho, amhCopula-method (assocMeasures), [21](#)
- rho, ANY-method (assocMeasures), [21](#)
- rho, claytonCopula-method (assocMeasures), [21](#)
- rho, copula-method (assocMeasures), [21](#)
- rho, evCopula-method (assocMeasures), [21](#)
- rho, fgmCopula-method (assocMeasures), [21](#)
- rho, frankCopula-method (assocMeasures), [21](#)
- rho, galambosCopula-method (assocMeasures), [21](#)
- rho, gumbelCopula-method (assocMeasures), [21](#)
- rho, huslerReissCopula-method (assocMeasures), [21](#)
- rho, indepCopula-method (assocMeasures), [21](#)
- rho, lowfhCopula-method (assocMeasures), [21](#)
- rho, mixCopula-method (mixCopula-class), [138](#)
- rho, moCopula-method (assocMeasures), [21](#)
- rho, nacopula-method (assocMeasures), [21](#)
- rho, normalCopula-method (assocMeasures), [21](#)
- rho, plackettCopula-method (assocMeasures), [21](#)
- rho, rotCopula-method (assocMeasures), [21](#)
- rho, tawnCopula-method (assocMeasures), [21](#)
- rho, tCopula-method (assocMeasures), [21](#)
- rho, tevCopula-method (assocMeasures), [21](#)
- rho, upfhCopula-method (assocMeasures), [21](#)
- rho-methods (assocMeasures), [21](#)
- rK (K), [121](#)
- rLatinHypercube (varianceReduction), [208](#)
- rlog, [182](#)
- rlogR (rlog), [182](#)
- rMvdc (Mvdc), [145](#)
- rmvdc (Mvdc), [145](#)
- rnacModel, [183](#), [185](#)
- rnacopula, [6](#), [180](#), [181](#), [184](#), [184](#), [186](#)
- rnchild, [184](#), [185](#), [185](#)
- rotCopula, [40](#), [126](#), [136](#), [187](#)
- rotCopula-class (rotCopula), [187](#)
- rownames, [202](#)
- rSibuya, [180](#), [181](#)
- rSibuya (Sibuya), [199](#)
- rSibuyaR (Sibuya), [199](#)
- RSpobs, [189](#)
- rstable, [193](#)
- rstable (rstable1), [191](#)
- rstable1, [179](#), [191](#)
- rtrafo (cCopula), [25](#)
- rug, [174](#), [175](#)
- safeUroot, [63](#), [64](#), [193](#)
- serialIndepTest, [116](#), [142–144](#), [195](#)
- serialIndepTestSim (serialIndepTest), [195](#)
- setClassUnion, [120](#)
- setTheta, [12](#), [36](#), [92](#), [97](#), [139](#), [197](#)
- setTheta, acopula, ANY-method (setTheta), [197](#)
- setTheta, copula, ANY-method (setTheta), [197](#)
- setTheta, ellipCopula, ANY-method (setTheta), [197](#)
- setTheta, khoudrajiCopula, ANY-method (setTheta), [197](#)
- setTheta, mixCopula, ANY-method (setTheta), [197](#)
- setTheta, outer\_nacopula, numeric-method (setTheta), [197](#)
- setTheta, Xcopula, ANY-method (setTheta), [197](#)

- show, [172](#), [199](#)
- show, acopula-method (acopula-class), [10](#)
- show, fitCopula-method (show-methods), [199](#)
- show, fitMvdc-method (show-methods), [199](#)
- show, interval-method (interval-class), [120](#)
- show, mvdc-method (mvdc-class), [147](#)
- show, nacopula-method (printNacopula), [172](#)
- show, normalCopula-method (show-methods), [199](#)
- show, parCopula-method (show-methods), [199](#)
- show, tCopula-method (show-methods), [199](#)
- show-methods, [199](#)
- Sibuya, [6](#), [199](#)
- sinc (math-fun), [133](#)
- SMI.12, [201](#)
- spearmansRho (assocMeasures), [21](#)
- splinefun, [123](#)
- splom, [156](#), [165](#), [203](#), [204](#)
- splom2, [156](#), [165](#)
- splom2 (splom2-methods), [203](#)
- splom2, Copula-method (splom2-methods), [203](#)
- splom2, data.frame-method (splom2-methods), [203](#)
- splom2, matrix-method (splom2-methods), [203](#)
- splom2, mvdc-method (splom2-methods), [203](#)
- splom2-methods, [203](#)
- Stirling, [204](#)
- Stirling1, [23](#)
- Stirling1 (Stirling), [204](#)
- Stirling2 (Stirling), [204](#)
- stop, [198](#)
- summary, [82](#), [90](#)
- summary, fitCopula-method (fitCopula-class), [85](#)
- summary, fitMvdc-method (fitCopula-class), [85](#)
- Summary, interval-method (interval-class), [120](#)
- summaryFitCopula-class (fitCopula-class), [85](#)
- summaryFitMvdc-class (fitCopula-class), [85](#)
- suppressWarnings, [89](#), [90](#)
- system.time, [148](#)
- tailIndex (assocMeasures), [21](#)
- tau, [5](#), [18](#), [170](#)
- tau (assocMeasures), [21](#)
- tau, acopula-method (assocMeasures), [21](#)
- tau, amhCopula-method (assocMeasures), [21](#)
- tau, ANY-method (assocMeasures), [21](#)
- tau, archmCopula-method (assocMeasures), [21](#)
- tau, claytonCopula-method (assocMeasures), [21](#)
- tau, copula-method (assocMeasures), [21](#)
- tau, evCopula-method (assocMeasures), [21](#)
- tau, fgmCopula-method (assocMeasures), [21](#)
- tau, frankCopula-method (assocMeasures), [21](#)
- tau, galambosCopula-method (assocMeasures), [21](#)
- tau, gumbelCopula-method (assocMeasures), [21](#)
- tau, huslerReissCopula-method (assocMeasures), [21](#)
- tau, indepCopula-method (assocMeasures), [21](#)
- tau, joeCopula-method (assocMeasures), [21](#)
- tau, lowfhCopula-method (assocMeasures), [21](#)
- tau, moCopula-method (assocMeasures), [21](#)
- tau, nacopula-method (assocMeasures), [21](#)
- tau, normalCopula-method (assocMeasures), [21](#)
- tau, plackettCopula-method (assocMeasures), [21](#)
- tau, rotCopula-method (assocMeasures), [21](#)
- tau, tawnCopula-method (assocMeasures), [21](#)
- tau, tCopula-method (assocMeasures), [21](#)
- tau, tevCopula-method (assocMeasures), [21](#)
- tau, upfhCopula-method (assocMeasures), [21](#)
- tau-methods (assocMeasures), [21](#)
- tauAMH, [206](#)
- tauJoe (tauAMH), [206](#)
- tawnCopula, [66](#)
- tawnCopula (evCopula), [65](#)
- tawnCopula-class (evCopula-class), [67](#)
- tCopula, [40](#), [48](#), [50](#), [92](#), [134](#), [135](#)

tCopula (ellipCopula), 46  
tCopula-class (ellipCopula-class), 50  
tevCopula, 66  
tevCopula (evCopula), 65  
tevCopula-class (evCopula-class), 67  
toEmpMargins (empCopula), 56  
toeplitz, 47  
trellis.par.get, 203  
TRUE, 81  
txtProgressBar, 100, 103, 106, 115, 142,  
144, 195, 213

uniroot, 13, 26, 64, 121, 122, 193, 194  
upfhCopula, 78  
upfhCopula (fhCopula), 78  
upfhCopula-class (fhCopula-class), 79  
uranium, 207

validObject, 139  
varianceReduction, 208  
vcov, 82, 86  
vcov.fittedMV (fitMvdc), 88  
vector, 81, 134, 135, 158, 159, 167, 190, 208  
vignettes, 6

warning, 80  
wireframe, 211, 212  
wireframe2 (wireframe2-methods), 211  
wireframe2, Copula-method  
    (wireframe2-methods), 211  
wireframe2, data.frame-method  
    (wireframe2-methods), 211  
wireframe2, matrix-method  
    (wireframe2-methods), 211  
wireframe2, mvdc-method  
    (wireframe2-methods), 211  
wireframe2-methods, 211

Xcopula-class (copula-class), 40  
xcopula-class (copula-class), 40  
xvCopula, 213